The Macroeconomic Consequences of Competition for College Admissions*

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Abstract

To study the role of competition for college admissions in driving parental investments and its implications for child development policies, we develop a heterogeneous-agent life-cycle model where parents invest in children's human capital to compete for limited college seats. The human capital acquired for college admission, however, only partially translates into labor efficiency units. Estimating our model with Chinese data, we find that competition drives 60% of parental investments on average, with a particularly large proportion for children near the admission threshold. Regulating competition through a private education investment tax balances curbing competition and minimizing human capital losses, improving welfare.

Keywords: Education Competition, Parental Investment, Human Capital, Intergenerational Mobility, Inequality, Childhood Development

JEL Classification: D31, D91, E21, J13, J24, J62, J68

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1 Introduction

Many countries in the world, regardless of being advanced or emerging economies, encounter a bottleneck in college admissions as a result of the limited availability of high-quality college seats and the high demand for such education. This leads to intense competition among prospective students. Parents are often compelled to invest substantial resources to increase their children's chances of securing admission, especially at elite universities.[1] Observing others' heavy investments, parents may follow suit out of concern for their children falling behind. Such competition incentive goes beyond investing to enhance children's human capital for being productive in future labor markets.

However, a pressing concern emerges from this situation: the human capital acquired to gain college admission may not effectively translate into the kind of productive human capital needed for success in the workforce. When the conversion rate is low, meaning that investing in human capital is mainly about competing for limited college seats and does not considerably improve labor efficiency as adult workers, the competition incentive can generate substantially excessive investment. In addition, it could disproportionately hurt children with disadvantaged backgrounds in terms of ability and family resources, exacerbating existing inequalities and hinder social mobility.

In this paper, we aim to address three key questions. First, to what extent does the competition for college admissions drive parental investment? Second, how does this affect children from disadvantaged family backgrounds, and what are the corresponding implications for inequality and mobility? Third, what are the welfare consequences, and how do those consequences affect policy design on child development when the competition incentive is prevalent?

We show that incorporating competition for scarce college seats into a model of child development creates an additional incentive for parental investment, alters significantly the effective returns on investment, and leads to distinct welfare consequences and policy conclusions. We demonstrate that the relative strength of the competition incentive depends crucially on the conversion rate from human capital acquired for college admission into labor efficiency units, and provide a novel identification strategy to gauge the rate. When a significant portion of parental investment is driven by the competition incentive, private returns on investment can far exceed social returns. In such cases, regulating competition through private education investment tax can improve welfare.

[1] For example, in the U.S., Kaushal, Magnuson, and Waldfogel (2011) find that families in the top expenditure quintile allocate 9% of their total expenditures on educational enrichment items. In India, selectivity at elite institutions prompt families to hire agents and tutors to boost their children's admission chances, which fuels an industry worth $35 billion (Frayer and Pathak, 2019).

We develop our findings using a general equilibrium (GE) heterogeneous-agent incomplete-market model (Huggett, 1993, 1996; Aiyagari, 1994; Rios-Rull, 1996) with realistic life cycles and parental investments to build children's human capital (Lee and Seshadri, 2019; Yum, 2023; Daruich, 2022). Specifically, children's pre-college human capital evolves through a multi-period production technology, where the next-period human capital is a result of the current-period human capital, parental monetary investments,[2] and public expenditures.

We incorporate two deviations into the model. The first deviation introduces competition for limited college seats—where the likelihood of admission purely depends on children's pre-college human capital and relative ranking. This generates a competition incentive for parents to invest in their children's pre-college human capital beyond the standard incentive that pre-college human capital could translate into labor efficiency units. The second deviation involves the partial conversion of pre-college human capital into productive human capital. Adult earnings are determined by an education-specific GE wage rate multiplied by labor efficiency units, derived from productive human capital, subject to idiosyncratic shocks. We allow the conversion rate to vary with respect to the stock of pre-college human capital.

We use data from the China Family Panel Studies (CFPS) on parental investments and child skills to discipline our model, as China serves as an ideal economic and institutional setting to address questions we seek to answer for three primary reasons. First, unlike the United States, where college admission success depends on various factors, China's college admission system features a highly simple procedure, with exam performance as the sole criterion for admission. This allows us to clearly identify the college competition incentive and estimate a human-capital-based college admission probability function. Second, China's college admission system is characterized by exceptionally intense competition, attributed to generous government subsidies for tuition and the high value parents place on their children's education.[3] This results in a significant disparity between the demand for college education and the limited available capacity, making the competition incentive particularly relevant and important. Third, because of the exam-oriented college admission system, parents spend significant amounts of money on private tutoring services to improve their children's ability to excel in exams relative

[2]Our focus on monetary investment stems from an emphasis on private tutoring, which is primarily a financial concern rather than demanding time input from parents. Throughout this paper, the term "parental investment" refers exclusively to parental monetary investment.

[3]This cultural emphasis on education, often referred to as "education fever," drives parents to invest considerable time, effort, and financial resources to ensure that their children receive the best possible education and opportunities to excel academically (Seth, 2002; Chen et al., 2020).

to their peers.[4] The heavy investment in cultivating this singe ability, overlooking that success in the labor market requires a diverse range of skills, may lead to a low conversion rate from human capital acquired to gain college admission to productive human capital. This necessitates government intervention. In 2021, the Chinese government decided to shut down the entire private tutoring sector, reflecting concerns about the potential negative impacts of college admission competition.

We estimate the model through a two-step strategy. We measure pre-college human capital utilizing the cognitive ability test scores for both adults and children. The first step estimates a set of parameters outside the model, including the college admission policy based on pre-college human capital. We find that the college admission probability is approximately an increasing convex function of individuals' pre-college human capital (normalized by the average) at the time of their college entrance examination. A sensitive region exists in which a small increase in pre-college human capital results in a substantial rise in the college admission probability.

In the second step, we use the method of simulated moments (MSM) to calibrate the remaining parameters. Our model matches data on parental monetary investment dynamics, child skill development, and other household characteristics. Because the child skill production function cannot be directly estimated, we utilize the data by fitting misspecified auxiliary models of the child skill formation technology and match those to their model counterparts to achieve indirect identification.

The most important parameter is the one governing the conversion rate from pre-college human capital into productive human capital. We offer a novel identification strategy that leverages the non-monotonicity of parental monetary investment with respect to child ability, a distinct empirical pattern observed in Chinese data, to identify the parameter. Our rationale is as follows. If the conversion rate is low, and diminishes as pre-college human capital increases, then a substantial rise in parental investment for children with abilities near the college admission threshold should imply that parents primarily invest in pre-college human capital to secure college admission rather than enhance labor efficiency. This is because parents anticipate a greater chance of their child gaining college admission, and even a small increase in investment could reap significant benefits by elevating their child's expected wage rate from a non-college to a college level. However, once a child's pre-college human capital is high enough to secure college admission, parents would not invest as much relative to their income. We find

[4] For example, in urban China, households allocate 12% of their earnings toward private tutoring, which corresponds to 15% of total household spending. This considerable investment fuels a thriving private tutoring industry worth over $150 billion.

pre-college human capital converts to productive human capital in a decreasing returns to scale manner with the scale parameter equal to 0.46.

Armed with a well-fitted model, we use it to quantify the importance of the competition incentive in driving parental investment and the subsequent outcomes, including sources of lifetime inequality. We perform this analysis by eliminating the competition incentive and comparing the results with the benchmark economy.[5] First, we find that for an average household, more than 60% of the parental investment is driven by the competition incentive, with the proportion being particularly large for children with marginal abilities (i.e., pre-college human capital levels close to the sensitive region of the college admission probability function).[6] Second, the competition incentive prompts low-income parents to prioritize investing in their children, even at the cost of their consumption, when their children possess marginal abilities. These findings suggest that reducing the competition incentive through policies could potentially lead to non-trivial welfare gains. Moreover, the conversion rate of pre-college human capital into productive human capital governs the relative strength of the competition incentive. As the rate increases, the competition incentive drives a smaller portion of parental investment.

The importance of the competition incentive also has key implications for sources of lifetime inequality. Consistent with previous studies in the U.S. context (e.g., Huggett, Ventura, and Yaron (2011) and Lee and Seshadri (2019)), our model also predicts that a significant portion (51%) of lifetime inequality is driven by the initial conditions predetermined before entering the labor market. Our paper differs from existing ones in that we find that when the competition incentive is prevalent, *nature*, or child innate ability, plays a more crucial role in driving lifetime income inequality compared to *nurture*, or parenting (28% vs. 15%). In contrast, studies on the U.S. economy with high-tuition college admission schemes have found that parental background, particularly income, has a larger explanatory power over individuals' pre-labor market conditions. We further corroborate this result by comparing our benchmark economy to an alternative one featuring a high-tuition college admission scheme, where *nature* accounts for only 11% of lifetime income variations, while college tuition affordability contributes 24%.

We then quantitatively evaluate the aggregate, welfare, and distributional effects of regulating parental investment competition in college admissions from both positive

[5] We eliminate the competition incentive in three ways: (1) relaxing the college capacity constraint; (2) drawing a lottery for college admission, ensuring that every child has an equal chance, regardless of their characteristics; and (3) increasing college costs such that the share of individuals who value college more exactly equals college capacity. All three scenarios exhibit highly similar parental investment patterns.

[6] More specifically, for a child in the first quintile of the pre-college human capital distribution, only around 15% of the investment is driven by the competition incentive. However, for a child in the fourth quintile, more than 70% of the investment is motivated by the competition incentive.

and normative perspectives. We begin by analyzing the impacts of China's 2021 private tutoring ban. We implement this policy by setting a cap on parental investment beyond which investment would not be allowed.

In the short run, for an average household, private investment decreases by approximately 39%, and as a result, children's pre-college human capital declines by about 6%. Welfare, measured in terms of the consumption-equivalent utility of the parent generation, increases by 0.5%. Regarding the distributional welfare effects, families with the lowest parental income and sufficiently high child innate ability benefit most from the private tutoring ban. These parents no longer need to sacrifice their consumption to invest as they did before, leading to the most substantial increase in their welfare. In contrast, families with the highest parental income and marginal child innate ability lose the most. Since high-income parents are prohibited from investing further to compete with high-ability children from low-income families, their children cannot secure college admissions as they did before the regulation.

In the long run, the private tutoring ban leads to a 0.6% decline in welfare, in contrast to an increase of 0.5% in the short run. Although reduced spending on children's education can increase parents' consumption and welfare, a reduction in children's human capital today results in them becoming less productive parents in the future, creating persistent human capital losses in the long run.

We finally investigate whether a policy can alleviate distortions from college admission competition without adversely affecting future generations. We restrict our policy instruments to a linear tax imposed on the private tutoring expenditure combined with a linear subsidy on pre-college public expenditures, financed solely by private tutoring tax revenue. We identify a 30% tax rate that maximizes long-term ex ante lifetime utility. The optimal tax balances curbing the competition incentive, which leads to inefficient excessive investment, and minimizing human capital losses as a result of reduced investment. Compared to a private tutoring ban, taxing private education investment significantly improves outcomes. The average pre-college human capital and lifetime earnings experience only a slight decline, whereas the welfare increases by 0.2%. Moreover, children with disadvantaged backgrounds, specifically those with low innate abilities and low-income parents, receive more investment from both parents and the government. Consequently, lifetime income inequality declines by 5%, and intergenerational persistence in lifetime income declines by 14%. We further show that both the optimal tax rate and welfare gains increase as the conversion rate declines, indicating a greater need for government regulation of parental investment competition.

Related Literature This paper contributes to several strands of literature. The first pertains to incorporating parental investments into a quantitative life-cycle model to study child human capital development with key implications on inequality, welfare, and policy. This strand of work highlights various aspects, including credit constraints and uninsured labor market risk (Lee and Seshadri, 2019; Caucutt and Lochner, 2020; Abbott, 2022), household composition (especially maternal labor supply) (Del Boca, Flinn, and Wiswall, 2014; Mullins, 2022), information frictions and inaccurate parental beliefs (Cunha, Elo, and Culhane, 2013), neighborhood effects (Fogli and Guerrieri, 2019; Agostinelli, Doepke, Sorrenti, and Zilibotti, 2022, Forthcoming; Chyn and Daruich, 2023), and the long-run impacts of early childhood development policies (Restuccia and Urrutia, 2004; Daruich, 2022; Füchs-Schundeln, Krueger, Ludwig, and Popova, 2023). We develop our model based in particular on Lee and Seshadri (2019), Yum (2023), and Daruich (2022). Compared to previous work, we incorporate an additional incentive for parental investment driven by competition for scarce college seats. We use the non-monotonicity of parental investment with respect to child ability to identify the conversion rate of human capital acquired for college admission into productive human capital, and rely on it to quantify the relative strength of the competition incentive. We show that accounting for such competition incentive has crucial policy implications for child development.

While existing literature focuses on addressing underinvestment due to liquidity constraints faced by young parents and market incompleteness, the competition incentive can lead to overinvestment, particularly for children with marginal abilities, as individual parents do not internalize the impact of their investments on others' investments. We show that when the competition incentive drives a signifcant portion of parental investments, regulating competition by taxing private education investment can be welfare-improving.

The second strand of literature examines education competition and its macroeconomic implications. These include the varying childcare time spent by parents with different education levels in the U.S. (Ramey and Ramey, 2010), low fertility and childlessness in South Korea (Kim, Tertilt, and Yum, forthcoming), the role of child efforts (Kang, 2022), and innovation and growth (Celik, 2023). Our study distinguishes itself by structurally identifying the sources of status externality discussed in Kim, Tertilt, and Yum (forthcoming), and, even more importantly, by decomposing the incentives driving parental investment into two components: the standard incentive to enhance a child's "real" human capital and thus labor productivity, and the competition incentive to secure college admission and enjoy college wage profiles. We demonstrate that the relative strength of the competition incentive determines the government's needs to regulate the competition, thus dictating the optimal tax rate on private education investment.[7]

Finally, our work contributes to the literature on the role of college and parental investments in shaping income inequality and social mobility (Becker and Tomes, 1979, 1986; Cunha et al., 2006; Cunha and Heckman, 2007; Caucutt and Lochner, 2020; Blandena, Doepke, and Stuhler, 2023), particularly how heterogeneous family backgrounds affect parental investments (Guryan, Hurst, and Kearney, 2008; Kaushal, Magnuson, and Waldfogel, 2011; Caucutt, Lochner, and Park, 2017; Caucutt, Lochner, and Mullins, 2023),[8] as well as children's college education outcomes in terms of admission and completion (Lochner and Monge-Naranjo, 2011; Bailey and Dynarski, 2011; Corak, 2013; Chetty et al., 2017; Cai and Heathcote, 2021; Blandin and Herrington, 2022; Capelle, 2020; Dudareva, 2022). These studies predominantly focus on the U.S. context, where high tuition fees make parental income a central factor in analyzing children's achievements and educational attainment. Policymakers' concerns primarily revolve around the potential inaccessibility of college education for low-income families, or insufficient parental investments for children from such backgrounds.

In this paper, we examine a distinct college admission model, prevalent in many Asian and European countries, where admission success primarily depends on child ability as measured by test scores. We show that under such an exam-oriented college admission scheme with limited college capacity, where the competition incentive is prevalent, parents, regardless of income levels, only invest significantly in children with sufficiently high abilities. Consequently, low-ability children may persistently face disadvantages due to limited private investment. Therefore, child ability could be another important factor in analyzing achievement and educational attainment. This finding may lead to different implications for understanding inequality, intergenerational persistence, and policy interventions aimed at reducing inequality and promoting mobility.

2 Model

We introduce a quantitative heterogeneous-agent incomplete-market model (Huggett, 1993; Aiyagari, 1994) with realistic life cycles $\grave{a} la$ Rios-Rull (1996) and Huggett (1996) and altruistic parents caring about their descendants' utility as described by Barro and Becker (1989), in an overlapping generations context. This model comprises three agent types:

[7]The tournament models discussed in Section 8 of Kim, Tertilt, and Yum (forthcoming) and also in Kang (2022) do not differentiate between the two types of human capital. This may affect the estimates on the effective return of parental investments and the quantitative importance of government intervention.

[8]For example, Kaushal, Magnuson, and Waldfogel (2011) and Caucutt, Lochner, and Park (2017) show that parental investments in children are strongly increasing in family income.

heterogeneous households, a representative firm, and the government. Households exhibit heterogeneity in various dimensions, including pre-college human capital, assets, education, and age. Adult earnings are determined by an education-specific general equilibrium wage rate multiplied by labor efficiency units, subject to idiosyncratic shocks, which cannot be fully insured. Households face borrowing constraints in each period and across generations, as parents are not allowed to borrow against their descendants' income. Young parents, based on their children's ability, decide how much money to invest in their children across multiple stages to develop pre-college human capital, alongside standard consumption-savings decisions.

The model features two deviations from frameworks used in the existing macroeconomic literature on child human capital development (Lee and Seshadri, 2019; Yum, 2023; Daruich, 2022). First, it introduces competition for limited college seats—where the likelihood of admission purely depends on a child's pre-college human capital and relative ranking. Such a college admission system gives parents an additional incentive to invest in their children's skills, beyond the standard incentive discussed in existing literature (e.g., Restuccia and Urrutia (2004); Lee and Seshadri (2019); Caucutt and Lochner (2020); Daruich (2022)), which is securing a college spot to elevate their children's wage profile from a non-college to college level. Second, it involves the partial conversion of pre-college human capital—invested in by parents to determine their children's chances of college admission—into productive human capital that determines labor efficiency units as adult workers. We allow the conversion rate to vary with respect to the stock of pre-college human capital.

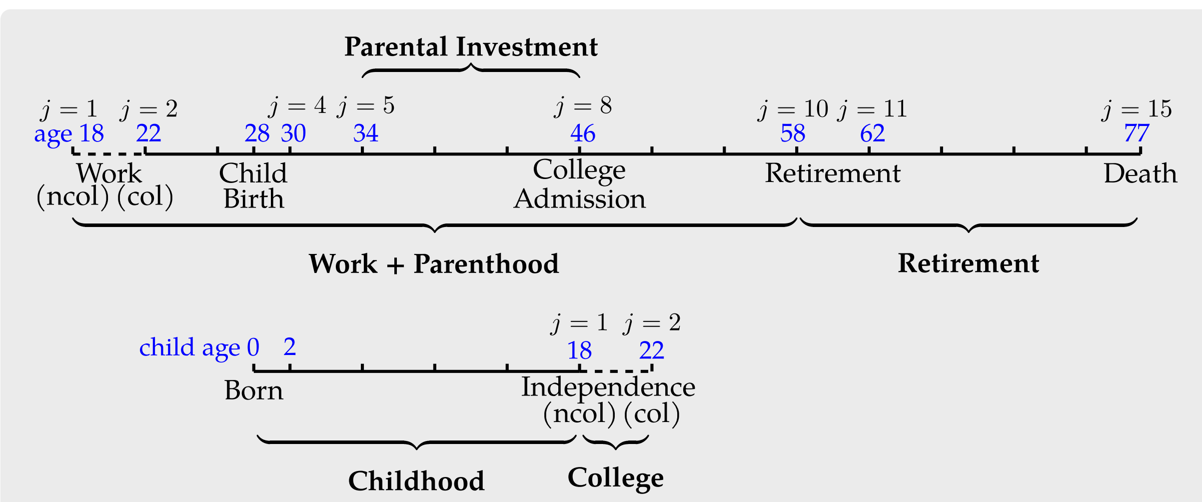

Figure 1: Timeline of the Model

2.1 Demographics and Environment

Time in the model is discrete, with one period corresponding to four years. A household consists of an adult living with a child until she becomes independent. Before a child becomes independent, she does not make any choices. The model follows a dynastic framework with four stages: childhood, college, work/parenthood, and retirement. Figure 1 summarizes the life-cycle events for a sample parent and child. Let j denote age in periods ($j = 1$ refers to ages 18-21, $j = 2$ to ages 22-25, etc.). The adult agent becomes independent and enters the labor market at either period $j = 1$ or $j = 2$ (age 18 or 22), working until retirement at periods $j = 10$ or $j = 11$ (age 58 or 62), depending on whether or not she attends college.[9] They then live for four periods after retirement and die at the end of period $j = 15$ (age 77).

In all periods, the adult agent makes consumption-savings choices, subject to uninsurable idiosyncratic labor productivity shocks. They save through a non-state-contingent asset. The child is born when the adult agent reaches the age of 28 (child age 0), corresponding to the middle between periods $j = 3$ and $j = 4$. The fertility is deterministic such that each individual parent has only one child. Parents start to invest in their children from the beginning of the period $j = 5$ (adult age 34-37, child age 6-9) until the end of the period $j = 7$ (just before a child reaches the age 18). At the beginning of $j = 8$ (child's age 18), parents make college decisions for their children based on the child's pre-college human capital as well as household income and wealth. If the value of attending college surpasses that of not attending, they will send their children to the college entrance examination, where the probability of college admission depends solely on the child's pre-college human capital relative to others in the same cohort. Newly formed households follow the same lifetime structure.[10] There is no population growth and no survival risk. We summarize the individual state variables in Table 1.

2.2 Wage Income Process

An individual's wage income depends on their education $s \in \{col, ncol\}$ (col stands for college and $ncol$ for non-college) and efficiency units y. Specifically, y encompasses the education-specific deterministic age profile A^s_j, productive human capital h_k, and

[9]This assumption also roughly captures the different retirement ages of blue-collar and white-collar workers averaged by gender in China.

[10]Our timeline specified is highly consistent with reality. According to the National Bureau of Statistics of China, the average age of first marriage in 2020 was around 26 years for women and 28 years for men. As for the average life span, data from the World Bank indicate that the life expectancy in China in 2020 was approximately 77 years.

Table 1: Individual State Variables

State variable	Description
j	Model age $j \in \{1, \ldots, 15\}$
s	Education of adults $s \in \{col, ncol\}$
a	Assets
h_p	Pre-college human capital of adults
z	Idiosyncratic labor productivity shocks
h	Pre-college human capital of children
s_c	Education of children $s_c \in \{col, ncol\}$

idiosyncratic labor productivity shocks z. z is mean reverting and follows a Markov chain with states $z = \{z_1, z_2, ..., z_M\}$, stationary distribution μ_z, and transitions $\pi(z'|z) > 0$. More specifically, $\log z_j = \rho_z \log z_{j-1} + \epsilon_z,\ \epsilon_z \sim N(0, \sigma_z)$.

Productive human capital h_k is a function of an individual's pre-college human capital h_p, governed by parameter λ, as follows:

$$h_k = h_p^{\lambda} \tag{1}$$

In Equation (1), we adopt the functional form from Abbott, Gallipoli, Meghir, and Violante (2020), Abbott (2022), and Daruich (2022), where they interpret λ as the return to pre-college human capital or ability gradient. λ in our context captures how pre-college human capital translates into productive human capital.

We define the conversion rate from h_p to h_k as follows:

$$\frac{dh_k}{dh_p} = \lambda h_p^{\lambda-1} \tag{2}$$

Note the conversion rate varies with respect to h_p. We abstract our model from two other types of human capital accumulation: (1) human capital accumulated in college;[11] and (2) on-the-job human capital accumulation.[12]

[11] One potential concern is that college education may improve cognitive ability. However, in Chinese data, we find that, as shown in Figure 3(A.3), an individual's cognitive ability test scores peak at around age 18 (the age to take the college entrance examination) and decline afterward. This suggests that college education may not improve test ability (although it is likely to improve ability in other dimensions).

[12] Existing macro literature, both without endogenizing initial conditions (e.g., Keane and Wolpin (1997); Huggett, Ventura, and Yaron (2011)) and with endogenizing initial conditions (e.g., Lee and Seshadri (2019); Daruich (2022); Daruich and Kozlowski (2020)), has demonstrated that most of the variations in lifetime income is attributable to conditions present before entering the labor market. An empirical study by Guvenen et al. (2022) using a 57-year-long panel from US Social Security Administration also supports this conclusion.

2.3 Preferences

Households are risk-averse, have identical preferences over consumption c, supply labor inelastically, and discount the future by β. The preferences are represented by a standard log utility function:

$$u(c) = \log(c).$$

We model altruism *à la* Barro and Becker (1989), in which the agent cares about their descendents' utility.

2.4 Firm Sector

We assume there is a representative firm that solves a static profit maximization problem as follows:

$$\max_{H_{col}, H_{ncol}, K} \left\{ Y - w^{col} H_{col} - w^{ncol} H_{ncol} - (r + \delta + \iota) K \right\} \tag{3}$$

Production technology is $Y = AK^{\Omega} H^{1-\Omega}$, where A denotes aggregate productivity, K is aggregate capital input, Ω denotes capital share, and $H = \left[\phi H_{col}^{\psi} + (1-\phi) H_{ncol}^{\psi} \right]^{\frac{1}{\psi}}$ denotes aggregate labor input, which is a CES aggregator of college labor H_{col} and non-college labor H_{ncol}. The term ϕ captures skill (college labor) intensity, ψ captures the elasticity of substitution between college and non-college labor, and $(r + \delta + \iota)$ is the capital rental cost, where ι captures financial intermediation cost.

2.5 Government Sector

The centralized government establishes an admission policy given fixed college capacity ξ, which specified the fraction of children who can go to college in the same cohort, and determine who can be admitted to college based on the rankings of pre-college human capital at age 18 (or at the start of $j = 8$). In our model, this policy is manifested as follows. The government takes ξ as given and adjusts the shifter ζ in the college admission policy function $\chi(\cdot, \zeta)$ to ensure the college capacity constraint is satisfied in equilibrium, where $\chi(h, \zeta)$ is increasing in an individual's pre-college human capital h, specifying the admission probability of each level of h relative to the cohorts.[13]

[13] Our model resembles China's public college system in a parsimonious manner. The system centers on the annual *Gaokao*, a mandatory standardized test for higher education admission. Five key features characterize the system: (1) public colleges dominate, resulting in a standardized, government-regulated system; (2) low tuition fees due to substantial subsidies; (3) limited quotas for each major, increasing competition for top institutions; (4) centralized allocation, matching students' preferences and *Gaokao* scores with available spots; and (5) high-stakes examination, with the *Gaokao* significantly affecting students' future prospects, causing immense pressure and competition in their final high school years.

Moreover, the government collects tax revenues in terms of a proportional consumption tax at rate τ_c and provides subsidies for college students $\varsigma_{college}$ and public investment in children's pre-college human capital by age before the college stage $g_j, j \in \{5, 6, 7\}$. τ_c is pinned down in equilibrium, as discussed in Section 2.7, and $\varsigma_{college}$ and $g_j, j \in \{5, 6, 7\}$ are exogenously given parameters.

2.6 Recursive Problems

2.6.1 Working Stage without Children

During the working stage without children (starting from $j = 1$ for individuals who do not go to college or $j = 2$ for those who go to college), individual households face a standard life-cycle problem. Given the linear consumption tax τ_c, households make consumption-saving decisions based on education s, assets a, pre-college human capital h_p, and idiosyncratic shocks z:

$$v_j(s, a, h_p, z) = max_{c,a'} \left\{ \log(c) + \beta \mathbb{E}_{z'} \left[v^s_{j+1}(s, a', h_p, z') \right] \right\} \tag{4}$$

subject to

$$(1 + \tau_c)\, c + a' = (1 + r)a + w^s y(h_p, A^s_j, z), \quad c, a' \geq 0$$

where

$$\log y(h_p, A^s_j, z) = \lambda \log h_p + A^s_j + \log z_j$$

Note $\lambda \log h_p$ is equal to the natural log of productive human capital h_k, as specified in Equation (1), contributing to labor efficiency units y.

2.6.2 Working Stage with Children and Parental Investment

At the beginning of $j = 5$, a child is born with some pre-college human capital endowment h_5,[14] which is perceived as the child's *innate ability*. The child's human capital at the end of childhood is affected by parental monetary inputs i and government inputs g_j in periods $j = 5, 6, 7$, as well as their innate ability h_5. The human capital production technology captures how these inputs affect the whole process. Our modeling approach builds on the childhood skill formation literature (Cunha and Heckman, 2007; Cunha, Heckman, and Schennach, 2010; Heckman and Mosso, 2014) insofar as it holds that skill formation is a multi-stage process and that human capital at an early stage serves as an

[14]Note that the age index $j = 5$ used here refers to the age of an individual parent when they start to invest in their child's human capital, but h_5 means child pre-college human capital endowment.

input in producing human capital at a later stage.

Child pre-college human capital production function The next-period child's pre-college human capital depends on the current-period child's pre-college human capital h, parental investment i, and age-specific public education investment g_j that is homogenous across children with heterogeneous backgrounds:

$$h' = \theta_j \left[\alpha h^{\gamma} + (1-\alpha) I_j^{\gamma}\right]^{\frac{1}{\gamma}}, I_j = \left[\eta_j i^{\mu} + (1-\eta_j) g_j^{\mu}\right]^{\frac{1}{\mu}} \tag{5}$$

where θ_j is the age-specific efficiency of child skill production technology, α captures the relative input share of own pre-college human capital in current period h, or self-productivity, γ is the elasticity of substitution between h and the CES aggregation of private monetary investment and public investment I_j, where η_j denotes the age-specific relative share of private investment, and μ denotes the elasticity of substitution between private and public investment.

Child Innate Ability and Intergenerational Persistence of Skills A child's innate ability h_5 is stochastic but correlated with the parent's pre-college human capital h_p by parental education level, which is specified as follows:

$$h_5 = (1-\rho_h)\mu^s + \rho_h h_p + \epsilon_h^s, \;\; \epsilon_h^s \sim N(0, \sigma_h^s) \;\; s \in \{col, ncol\} \tag{6}$$

where ρ_h captures intergenerational persistence of pre-college human capital, μ^s is the average child pre-college human capital endowment with parental education s, and σ_h^s is the standard deviation of intergeneratinal genetic shocks for parental education s.

Recursive Problem with Parental Investment We assume that the child shares household consumption c and does not make any independent decisions relevant to the household's economic status during childhood. The following value function summarizes the decision problem of a parent with child pre-college human capital h for $j = 5, 6, 7$:

$$v_j(s, a, h_p, z, h) = max_{c,a',i} \left\{ \log(c) + \beta \mathbb{E}_{z'} \left[v_{j+1}(s, a', h_p, z', h') \right] \right\} \tag{7}$$

subject to

$$(1+\tau_c)\, c + a' + i = (1+r)a + w^s y(h_p, A_j^s, z), \; c, a', i \geq 0$$

together with Equation (5) and (6).

2.6.3 College Attendance Stage

At the beginning of period $j = 8$ (child age 18), a parent compares the value of going to college $v(\cdot, s_c = col)$ with that of not going to college $v(\cdot, s_c = ncol)$ for her child where $s_c \in \{col, ncol\}$ denotes education outcomes of children. If $v(\cdot, col)$ surpasses $v(\cdot, ncol)$, parents send their children to take the college entrance examination. For a child with $v(\cdot, col) > v(\cdot, ncol)$, their admission outcome relies solely on their pre-college human capital at the beginning of period $j = 8$, h_8, relative to that of other children, which is governed by the college admission probability function $\chi(\cdot, \zeta)$. Note that we abstract our model from inter vivos transfers,[15], as well as the option to go to college overseas, as this alternative is mainly accessible to privileged families and represents a very small portion of the total population.[16] Given $\chi(\cdot, \zeta)$, individual parents solve the following problem:

$$\begin{aligned} v_8(s, a, h_p, z, h) = & \ v_8(s, a, h_p, z, h, ncol)\mathbb{1}_{v(\cdot,col)\leq v(\cdot,ncol)} \\ & + \chi(h, \zeta) v_8(s, a, h_p, z, h, col)\mathbb{1}_{v(\cdot,col)>v(\cdot,ncol)} \\ & + (1 - \chi(h, \zeta)) v_8(s, a, h_p, z, h, ncol)\mathbb{1}_{v(\cdot,col)>v(\cdot,ncol)} \end{aligned} \tag{8}$$

where the value of individual parents whose children do not go to college $v(\cdot, ncol)$ is:

$$v_8(s, a, h_p, z, h, ncol) = max_{c,a'} \left\{ \log(c) + \beta \underbrace{\mathbb{E}_{z'}\left[v_9(s, a', h_p, z')\right]}_{\text{Parent value}} + \nu \underbrace{\int v_1(ncol, 0, h, z)\mu_z(z)}_{\text{Child value}} \right\} \tag{9}$$

subject to

$$(1 + \tau_c)\, c + a' = (1 + r)a + w^s y(h_p, A^s_j, z), \ c, a' \geq 0.$$

and the value of agents whose children go to college $v(\cdot, col)$ is specified as:

$$v_8(s, a, h_p, z, h, col) = max_{c,a'} \left\{ \log(c) + \beta \underbrace{\mathbb{E}_{z'}\left[v_9(s, a', h_p, z')\right]}_{\text{Parent value}} + \nu\beta \underbrace{\int v_2(col, 0, h, z)\mu_z(z)}_{\text{Child value}} \right\} \tag{10}$$

[15]Numerous studies in the existing literature highlight the important role of inter vivos transfers in college education outcomes and intergenerational mobility in the U.S. (Abbott, Gallipoli, Meghir, and Violante, 2020; Daruich and Kozlowski, 2020). These transfers primarily serve to finance children's costly tuition and fees for attending American colleges. In contrast, in urban China, inter vivos transfers are scarce during the college decision-making stage, as parents directly cover their children's tuition and fees.

[16]For example, in 2019, around 1,194,900 Chinese students studied abroad (including all education levels), which is about 0.43% of the total urban population aged 7-25 in China (estimated at 280.07 million).

subject to

$$(1+\tau_c)\,c + a' + \kappa = (1+r)a + w^s y(h_p, A_j^s, z),\ c, a' \geq 0.$$

where ν captures parents' altruistic motives and κ denotes college tuition and fees paid by parents. Note that children become independent at age of 18 if they do not go to college, and at age of 22 if they go to college, so they have separate value function from their parents where they start with zero assets $a = 0$ and draw labor productivity shocks z from μ_z, and the state variable h no longer appears in parents' value function. For children who go to college, they spend four years in college, so they enter the labor market at model period $j = 2$. After the agent's child becomes independent, her individual problem is equal to (4).

2.6.4 Retirement Stage

At the beginning of periods $j = 11$ or $j = 12$ (age 58 for adults without college degree or age 62 for college-educated adults), the agent retires with the source of income coming from savings, and $j = 15$ being the end of the life cycle.[17] The recursive problem is:

$$v_j(a) = \max_{c,a'} \{\log(c) + \beta v_{j+1}(a')\} \tag{11}$$

subject to

$$(1+\tau_c)\,c + a' = (1+r)a,\ c, a' \geq 0$$

2.7 Stationary Equilibrium

Let $x_j \in X_j$ be the age-specific state vector of an individual of age j, as defined by the recursive representation of the individual household's problems in Section 2.6. Let the Borel sigma-algebras defined over those state spaces be $\Psi = \{\Psi_j, \Psi_h\}$. A stationary recursive competitive equilibrium for this economy is a collection of: (i) private households' decision rules for consumption, asset holdings, and parental investment $\{c_j(x_j), a_j'(x_j), i_j(x_j)\}$, and the associated value functions $\{v_j(x_j)\}$; (ii) a college admission shifter ζ set by the government in college admission policy function $\chi(\cdot,\zeta)$, and consumption tax τ_c; (iii) aggregate capital and labor inputs of the representative firm $\{H_{col}, H_{ncol}, K\}$; (iv) prices $\{w^{col}, w^{ncol}, r\}$; and (v) a vector of measures of Ψ such that:

1. Given prices, decision rules solve individual's problems (4), (7), (8), and (11);

[17]For simplicity, we abstract our set-up from social security tax and hence social security benefits provided by the government.

2. Given prices, aggregate capital and labor inputs solve the representative firm's problem (3);

3. The asset market clears:
$$K = \sum_{j=1}^{15} \int_{X_j} a'_j \left(x_j\right) d\Psi_j. \tag{12}$$

4. Labor markets clear for both education levels:
$$H_{col} = \sum_{j=2}^{11} \int_{X_j} y_j^{col} \left(x_j\right) d\Psi_j, H_{ncol} = \sum_{j=1}^{10} \int_{X_j} y_j^{ncol} \left(x_j\right) d\Psi_j \tag{13}$$

5. The college capacity constraint is satisfied:
$$\frac{\int_{X_8} \int_{H_h} \chi\left(h, \zeta\right) \mathbb{1}_{v_8(x_8,col)>v_8(x_8,ncol)} d\Psi_h d\Psi_8}{\int_{X_8} \Psi_8} = \xi \tag{14}$$
where H_h denotes the state space for pre-college human capital, Ψ_h denotes the distribution of the pre-college human capital of children at age 18 (corresponding to the beginning of parent period $j = 8$), and ξ denotes the fixed college capacity.

6. The government budget is balanced every period as follows:
$$\varsigma_{college} \int_{X_8} \int_{H_h} \chi\left(h, \zeta\right) \mathbb{1}_{v_8(x_8,col)>v_8(x_8,ncol)} d\Psi_h d\Psi_8 + \sum_{j=5}^{7} \int_{X_j} g_j d\Psi_j = \sum_{j=1}^{15} \int_{X_j} \tau_c c_j \left(x_j\right) d\Psi_j \tag{15}$$

7. Individual and aggregate behaviors are consistent: measure Ψ is a fixed point of $\Psi(X) = Q(X, \Psi)$ where $Q(X, \cdot)$ is a transition function generated by decision rules and exogenous laws of motion, and X is the generic subset of the Borel-sigma algebra defined over the state space.

2.8 Discussion on Key Model Mechanism

Our quantitative model provides a rich environment to investigate the impacts of competition for limited college seats on parental investment, child outcomes, inequality, mobility, and potential policy interventions. Before moving forward, we discuss the key mechanism of the quantitative model through the lens of a simple two-period model.

We demonstrate two key points below. First, the source of excessive investment arises from individual parents not internalizing the impact of their investment on the college admission threshold decided by the government. Second, the relative strength of the competition incentive for parental investment (and thus the degree of excessive investment) is governed by the conversion parameter λ given parental income and the college wage premium. Both points rationalize the necessity of government intervention in regulating parental investment competition in college admissions.

2.8.1 Simple Two-period Model

Consider a two-period economy with a continuum of heterogeneous households consisting of a parent and a child, as well as a government that decides who can be admitted to college given the inelastic supply of college seats. Households differ in two dimensions: parent income m and child innate ability h_o. Parents split their income between consumption c_1 and investment i in their children's pre-college human capital.

The pre-college human capital h is determined by skill production function $h = h_o^{\alpha} i^{(1-\alpha)}$.[18] Note that h plays two roles. First, children's income is an increasing function of h given the education level. Second, the probability of college admission and thus earning college wage is increasing in h. Being a college-educated worker means she can earn college wage income $w^{col}h^{\lambda}$. Otherwise, she earns non-college wage income $w^{ncol}h^{\lambda}$. λ governs the conversion rate of pre-college human capital to labor efficiency units.

The college admission probability is governed by $\chi(h, \zeta)$, which is a function of h and a shifter parameter ζ (capturing college admission threshold) set by the government. We assume $\chi(\cdot)$ is increasing and convex in h and decreasing in ζ.

Private households Given m, h_o, $\chi(h, \zeta)$, private households solve the following:

$$\max_{c_1, i} \left\{ \log(c_1) + \chi(h, \zeta) \log(c_2^H) + [1 - \chi(h, \zeta)] \log(c_2^L) \right\} \tag{16}$$

subject to

$$c_1 + i = m, \;\; h = h_o^{\alpha} i^{(1-\alpha)}$$

$$c_2^H = w^{col} h^{\lambda}, \;\; c_2^L = w^{ncol} h^{\lambda}$$

Simplifying the problem yields

$$\max_{i} \left\{ \log(m - i) + \chi(h, \zeta) \log(w^{col} \left(h_o^{\alpha} i^{(1-\alpha)}\right)^{\lambda}) + [1 - \chi(h, \zeta)] \log(w^{ncol} \left(h_o^{\alpha} i^{(1-\alpha)}\right)^{\lambda}) \right\} \tag{17}$$

[18] Note that the skill production function in this simple model takes the Cobb-Douglas functional form, which is nested within the skill production equation (Equation (5)) in our benchmark model.

Private First Order Condition Taking the first order condition (FOC) with respect to i gives the following Euler equation:

$$\underbrace{\frac{1}{1-\alpha} \cdot \frac{1}{m-i}}_{\text{MC}\,(m)} = \underbrace{\frac{\lambda}{i}}_{\text{standard incentive}} + \underbrace{\log\left(\frac{w^{col}}{w^{ncol}}\right)\left(\frac{h_o}{i}\right)^{\alpha} \frac{\partial \chi(h,\zeta)}{\partial h}}_{\text{competition incentive}} \tag{18}$$

Differentiating Two Types of Investment Incentives The left-hand side of Equation (18) represents the marginal cost of investing an additional unit, which primarily depends on parental income. For parents with higher income, the marginal cost is lower as they do not need to sacrifice significant consumption utility for increased investment in their children. The right-hand side of Equation (18) demonstrates the marginal benefit of an additional investment unit, comprising two components: (1) the *standard incentive*, where one more unit of investment yields λ units of return, conditional on the education level, and (2) the *competition incentive*, where an extra unit of investment increases the probability of college admission ($\frac{\partial \chi(h,\zeta)}{\partial h}$), thus enjoying college wage premium $\left(\frac{w^{col}}{w^{ncol}}\right)$. When h is close to the college admission threshold as a result of high innate ability h_o, the private return of investing one more unit of i can be very large, as even a small increase in investment could greatly increase the chance of elevating their children's earnings from a non-college to college level.

Government The college capacity is fixed at ξ. The government chooses ζ to make sure the college capacity constraint is satisfied. That is, $\int \chi(h,\zeta)d\varphi = \xi$ where φ denotes the distribution of children's pre-college human capital.

Social First Order Condition Suppose the government can decide investment for households characterized by $\{m, h_o\}$. The government internalizes the effect of individual household's investment i on college admission threshold ζ. Then the FOC with respect to i becomes:

$$\frac{1}{1-\alpha} \cdot \frac{1}{m-i} = \frac{\lambda}{i} + \log\left(\frac{w^{col}}{w^{ncol}}\right)\left(\frac{h_o}{i}\right)^{\alpha}\left(\frac{\partial \chi(h,\zeta)}{\partial h} + \frac{\partial \chi(h,\zeta)}{\partial \zeta}\frac{\partial \zeta}{\partial h}\right) \tag{19}$$

2.8.2 Insights from the Simple Model

Insight 1 *The source of excessive investment arises from individual parents not internalizing the impact of their investments on the college admission threshold decided by the government.*

Note that the right-hand side (RHS) of Equations (18) and (19) captures the private marginal benefit and social marginal benefit of investing one more unit of numeraire goods into a child with ability h_o, respectively. Subtracting the RHS of Equation (18) from Equation (19) generates the extra term $\log\left(\frac{w^{col}}{w^{ncol}}\right) h_o \frac{\partial \chi(h,\zeta)}{\partial \zeta}\frac{\partial \zeta}{\partial h}$. If every parent increases i such that h increases and the distribution of children's human capital shifts toward a higher level of h, then the government needs to increase admission threshold ζ to make sure the college capacity constraint is still satisfied. This means $\frac{\partial \zeta}{\partial h} > 0$. On the other hand, given h, the probability of getting into college $\chi(h, \cdot)$ is decreasing in ζ. This means $\frac{\partial \chi}{\partial \zeta} < 0$. Consequently, $\frac{\partial \chi(h,\zeta)}{\partial \zeta}\frac{\partial \zeta}{\partial h} < 0$.

In other words, when an individual household makes the decision, she takes the college admission shifter ζ determined by the government as given, but she does not internalize the impact of her private investment on the college admission shifter. Therefore, the source of excessive investment arises from the extra term $\log\left(\frac{w^{col}}{w^{ncol}}\right) h_o \frac{\partial \chi(h,\zeta)}{\partial \zeta}\frac{\partial \zeta}{\partial h} < 0$, which causes the social return on investment to be smaller than the private return.

Insight 2 *Given the college wage premium $\frac{w^{col}}{w^{ncol}}$ and parental income m, the relative strength of the competition incentive for parental investment (and thus the degree of excessive investment) is governed by the conversion parameter λ.*

We illustrate *Insight 2* through Private FOC Equation (18). Given the college wage premium $\frac{w^{col}}{w^{ncol}}$ and parental income m, if λ is very high, it implies that parents mainly invest in h to increase labor efficiency units h^{λ}. As a result, the competition incentive driving parental investments is weak. Take the other extreme where λ equals zero. Investing in h is solely about competing for a limited number of college seats and does not improve children's earnings except for obtaining the college wage rate w^{col}. In this case, the competition incentive is strongest. Similarly, given a fixed λ and parental income m, a higher college wage premium $\frac{w^{col}}{w^{ncol}}$ indicates a stronger competition incentive.

Equation (18) provides valuable insights on how to identify λ. Given the college wage premium $\frac{w^{col}}{w^{ncol}}$ and parental income m, if λ is large enough for the standard incentive to dominate the competition incentive, we would expect little variation in parental investment regardless of child ability h_o. On the other hand, if λ is very small, for instance, zero, the private return to investment i is highly non-linear. The return would be very low for the lowest level of h_o due to the slim chance of college admission, but it would dramatically increase for children with h close to the college admission threshold as a result of sufficiently high h_o. Consequently, we would observe that parental investment significantly increases for children with sufficiently high cognitive abilities in the data.

Throughout our analysis, we can see that identifying λ can be achieved by examining the patterns of parental investment dependence on child abilities while controlling for parental income and the college wage premium. This helps take our quantitative model to the data in Section 3.

3 Mapping the Model to the Data

In this section, we outline the process of mapping the model to the data. Our baseline calibration focuses on the life cycle of one generation of adults and one generation of children with fixed prices. This approach is motivated by the significant disparity in the college labor share between parent and child generations in Chinese data, with only 7.5% in the parent generation but 35% in the child generation, primarily driven by a large-scale college expansion reform.[19] This means when parents make investment decisions, they consider the college expansion reform, anticipating that their children will have a significantly higher likelihood of attending college compared to their generation. As a result, assuming a stationary distribution in general equilibrium is not feasible. To assess the long-run effects, we will switch to a general equilibrium life-cycle overlapping generations model with stationary household distributions, as defined in Section 2.7.

We discipline our model to using Chinese data through a two-step estimation strategy. In the first step, we employ data to estimate or calibrate a set of parameters that can be distinctly identified outside our model. In the second step, we apply the method of simulated moments (MSM) to estimate the remaining model parameters, matching it to data on the Chinese economy in the 2010s, in accordance with the availability of the data sources utilized in this paper.

3.1 Data

3.1.1 Data Sources

The primary data source for estimating our model is the China Family Panel Studies (CFPS), a nationally representative biennial longitudinal household and individual survey initiated in 2010 by the Institute of Social Science Survey at Peking University. Comparable to the Panel Study of Income Dynamics (PSID) in the United States, the CFPS tracks individuals across waves. As of February 2023, six waves of data had been released. The baseline CFPS comprises around 15,000 households and 30,000 individuals, with a response rate of 79%. Five provinces are initially oversampled, with 1,600 families

[19] In the late 1990s, China implemented a policy-driven large-scale college expansion reform. Consequently, the four-year college entrance rate has increased more than sixfold over the past two decades.

selected from each province to enable regional comparisons. The rest of the sample is drawn from other provinces to achieve national representativeness through weighting. Overall, the CFPS represents 95% of China's population.[20]

We consider the CFPS to be the best available dataset for our study for two main reasons. First, it measures cognitive ability, akin to the Armed Forces Qualification Test (AFQT) in the United States, for both adult and child respondents, which are our primary measures of pre-college human capital. Second, it contains adult (aged 16 and above) and child (aged 10-15) samples that enable the construction of parent-child pairs. This allows us to examine how parent background, in terms of income, education, and pre-college human capital, as well as child innate ability, influences parental investment in children and subsequent outcomes.

We supplement our analysis with other data sources, including the China Education Finance Statistical Yearbook (CEFSY), the Chinese Household Income Project (CHIP), the China Health and Nutrition Survey (CHNS), and the Urban Household Survey (UHS).

3.1.2 Main Variables

College We determine an individual's college education status using information on years of education and school level. Following the World Bank's definition, we consider a four-year tertiary degree or its equivalent (e.g., a bachelor's degree in the United States) or higher as college education.[21]

Income Our baseline income measure is labor income, which refers to income earned solely through salaries in the previous survey year, deflated using the 2010 Consumer Price Index (CPI).

Test Scores and Pre-college Human Capital The CFPS includes detailed cognitive ability assessments comparable to the AFQT scores in the United States. It has four types of cognitive tests: literacy, math, word recall, and numerical series. Each test assesses a different aspect of cognition, and the four types complement one another. The CFPS alternates two sets of these assessments across waves: Set A (literacy and math tests) measures educational achievement, while Set B (word recall and numerical series tests)

[20] See Xie and Lu (2015) and Xie and Zhou (2014) for a detailed discussion of the sampling design.

[21] Since our focus is on the parental investment competition for college admissions, we only consider more competitive four-year colleges, known as *Benke* in Chinese. We will not consider three-year colleges, referred to as *Zhuanke*. In recent years, the admission rate after accounting for three-year colleges in urban China has exceeded 85%.

reflects respondents' fluid intelligence. Set A was used in 2010, 2014, and 2018, and Set B in 2012, 2016, and 2020. We focus on the 2010, 2014, and 2018 waves featuring Set A to ensure consistent cognitive ability measurement. We measure pre-college human capital as the sum of math test and word test scores.[22] For the child survey, we can only observe children's cognitive ability scores starting at age 10 and must infer their cognitive ability score distribution before age 10 through extrapolation.

Parental Investment We focus on monetary investment made by parents. The CFPS data cover private education expenditures including school fees, extracurricular activities, private tutoring, books, boarding, and transportation fees.[23]

3.1.3 Sample Selection

We focus on the 2010, 2014, and 2018 waves that include cognitive ability scores. Additionally, we concentrate on urban households because of the prevalence of parental investment competition in college admissions in urban China. Our basline sample is restricted to households with one child under age 21 and parents under age 60. We exclude parent-child pairs with missing information on education, income, cognitive ability scores for either parent, education expenditures, and children's cognitive ability scores. When available, we use the father's information; otherwise, we rely on the mother's. This results in 8,126 parent-child pairs. Parents in our sample were born between 1965 and 1980, taking the college entrance examination before the 1999 college expansion reform. Children were born between 1993 and 2008, taking the exam after the reform. Consequently, the college labor share differs significantly between generations, as mentioned above: 7.5% in the parent generation and at least 35% in the child generation.[24]

3.2 Externally Calibrated Parameters

In the first step, we directly estimate four sets of parameters from the data, including: (1) pre-college human-capital-based college admission policy; (2) age profile of earnings by education levels; (3) parent pre-college human capital distributions by education levels;

[22]Alternatively, we could adjust scores by weighting each of the 50 questions by the inverse of the fraction of respondents who answered correctly, as in Lee and Seshadri (2019). This alternative does not considerably change our identification results.

[23]However, detailed categories of private education expenditures are only available in the 2010 wave. We supplement this information with data from the 2018 wave of the CHIP, which provides detailed categories of private education expenditure as the CFPS 2010.

[24]This number is computed based on the observable college outcomes of the child's generation in our CFPS sample, which aligns with 2020 Census of China.

and (4) public investment by the stage of pre-college education. We report the results on the externally estimated parameters in Table 2 and Figure 2. Note that we normalize the average household income to be one.

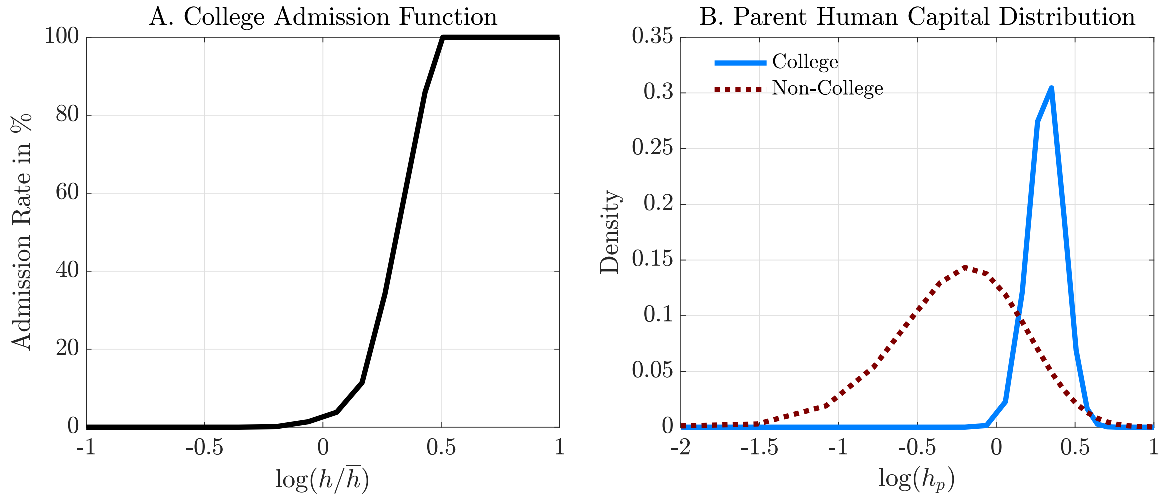

Figure 2: Pre-college Human-capital-based College Admission Policy

Pre-college human-capital-based college admission policy We estimate the pre-college human-capital-based college admission function using the adult survey of the CFPS, where we can observe the measure of pre-college human capital (i.e., cognitive ability scores) in both their early career and education outcomes. We restrict the age range to 24-35, as this allows us to observe both cognitive ability scores and education outcomes, with ages close enough to the college entrance examination so that cognitive ability scores primarily reflect pre-college human capital.[25] We first compute the mean of the cognitive ability scores for all individuals in our sample. Then, we divide the distribution of cognitive ability scores into 10 bins based on their rank from the lowest to the highest. For each bin, we compute the distance between the mean of cognitive ability scores of each bin and the mean of all individuals (denoted by $\bar{h}$) in logs, as well as the share of people with a college degree.[26] Here, we only care about the shape of the college

[25]Ideally, with a sufficiently long panel dataset (e.g., the PSID) with sufficiently large number of observations, we could observe individuals' early childhood development stages, adult education, and labor market outcomes. We would use their cognitive ability scores at age 18 and track their education outcomes to estimate the human-capital-based college admission probability function. However, since we have data from only three waves, we have to treat the dataset as cross-sectional. To mitigate potential issues, we select adults aged 24-35, not far from college graduation, minimizing changes in pre-college human capital (measured by cognitive ability scores) after entering the labor market.

[26]In other words, we do not differentiate between college admission and college completion, as the college completion rate in China is very high, often above 95% for each cohort.

admission function, which essentially dictates the probability of college attendance as a function of pre-college human capital.

We report the estimation results in Figure 2(A). We observe that the college admission probability is approximately an increasing convex function of a child's pre-college human capital (normalized by the average) at the time of their college entrance examination within the domain $[-1, 0.5]$.[27] A sensitive region emerges when a child's pre-college human capital is approximately 20% higher than the average, where a small increase in pre-college human capital results in a substantial rise in college admission probability.[28]

Firm Production Technology We set the capital share parameter $\Omega = 0.47$ and depreciation rate $\delta = 0.11$ following Bai, Hsieh, and Qian (2006). The initial iceberg $\iota = 0.07$ is set so that the gross rate of return to capital is 11% in the early 2010s (in line with the estimates of Chen et al. (2019)). The elasticity of substitution between college-educated and non-college-educated labor ψ is set to be 0.50 following Daruich (2022).[29]

Public Investments Public investment is a key input for child human capital production.[30] For this component, we rely on data from the CEFSY. Our data analysis reveals that pre-college public investment is roughly twice the private investment. Moreover, the public education expenditure corresponds to approximately 15% to 30% of the average household labor income (normalized to be one in the model), informing the value of g_j parameter. College tuition and fees roughly equal 17% of the average household labor income, helping us pin down the κ parameter in our model.[31] Finally, the government subsidy per college student $\varsigma_{college}$ is nearly three times higher than college tuition and

[27]Note that there are very few individuals in the data whose h is larger than $exp(0.5)\bar{h}$.

[28]Concerns may arise regarding the ability of a single college admission policy function to accurately represent regional disparities in China's college admission system, as students from more developed regions often have a better chance of being admitted to prestigious institutions because of the lower required entrance examination scores. However, it is important to emphasize that our measurement of pre-college human capital relies on standardized cognitive ability test scores, which are not influenced by regions and demonstrate robust predictive power for college admission probabilities.

[29]We use the estimate from Daruich (2022) because Daruich (2022) specifies college-educated workers as those who have completed four years of college, although the parameter is estimated using the U.S. data. Ge and Yang (2014) estimate the elasticity of substitution at 0.60 using the UHS data of China. However, their definition of college (both three- and four-year) is not consistent with the one (four-year college only) used in our paper. Nevertheless, these two estimations (i.e., 0.50 and 0.60) are quite close to each other.

[30]In China, the majority of children attend public schools. In 2022, public schools accounted for 89.4% of primary and junior high school students and 82.7% of senior high school students.

[31]In China, the average tuition fee for a four-year college is around 5,000 yuan, while accommodation fees are approximately 1,000 yuan annually. These amounts remain relatively consistent across schools, regardless of quality levels. The average earnings across the working-age population in the urban area in the data is around 36,000 yuan. Consequently, κ equals 6000/36000 = 0.17.

Table 2: Externally Estimated Parameters

Parameter		Value	Source/Target
(A) Production			
Capital share	Ω	0.47	Bai, Hsieh, and Qian (2006)
Depreciation rate	δ	0.11	Bai, Hsieh, and Qian (2006)
Iceberg cost	ι	0.07	Chen et al. (2019)
Elasticity of substitution (col. & non-col.)	ψ	0.50	Daruich (2022)
Aggregate productivity	A	1.00	Normalization
(B) Government policies			
Public education investment $j=5$	g_5	0.16	CEFSY
Public education investment $j=6$	g_6	0.23	CEFSY
Public education investment $j=7$	g_7	0.29	CEFSY
College tuition and fees	κ	0.17	CEFSY
College subsidy	$\varsigma_{college}$	0.52	CEFSY
Fixed college capacity	ξ	0.35	CFPS and Census
(C) Parent human capital and age profile			
Mean: Parent human capital dist. (college)	M^{col}	83.9	CFPS 2010-18
Mean: Parent human capital dist. (non-col.)	M^{ncol}	7.50	CFPS 2010-18
SD: Parent human capital dist. (college)	Q^{col}	0.02	CFPS 2010-18
SD: Parent human capital dist. (non-col.)	Q^{ncol}	0.13	CFPS 2010-18
Age profile of earnings by education	A_j^s	Figure A2	CHNS

fees paid by households.[32]

Initial Distribution of Parents We assume that adults (parent generation) begin with zero assets, and the initial distribution of parental pre-college human capital (h_p^s) follows an education-specific Gamma distribution:

$$h_p^s \sim \Gamma(M^s, Q^s), \quad s \in \{col, ncol\}.$$

We calculate the average and standard deviation of parental cognitive skills using the CFPS data, establishing the parameters that shape the Gamma distribution in our baseline model for calibration, as reported in Figure 2(B). Note that we calibrate the initial distribution of parental skills outside the model, without treating it as an endogenous outcome of the stationary equilibrium. As discussed before, since in the data, parent and child generations experienced their upbringing before and after the large-scale college

[32]See Table A1 in Appendix A for more details.

expansion reform, the human capital and education distributions for both cohorts display notable differences, which do not align with a stationary distribution.

3.3 Internally Calibrated Parameters

We summarize the internally calibrated parameters together with moments used to discipline them in Table 3. These moments encompass the estimated parameters of misspecified translog child skill production functions, age profiles of child cognitive ability and parental monetary investments, covariances between relevant data features, and income elasticity of investment. The 21 model parameters are determined by minimizing the distance between the set of empirical moments and their model counterparts. Although every targeted moment is influenced by all parameters, we discuss each in relation to the parameter that, intuitively, offers the most identification power.

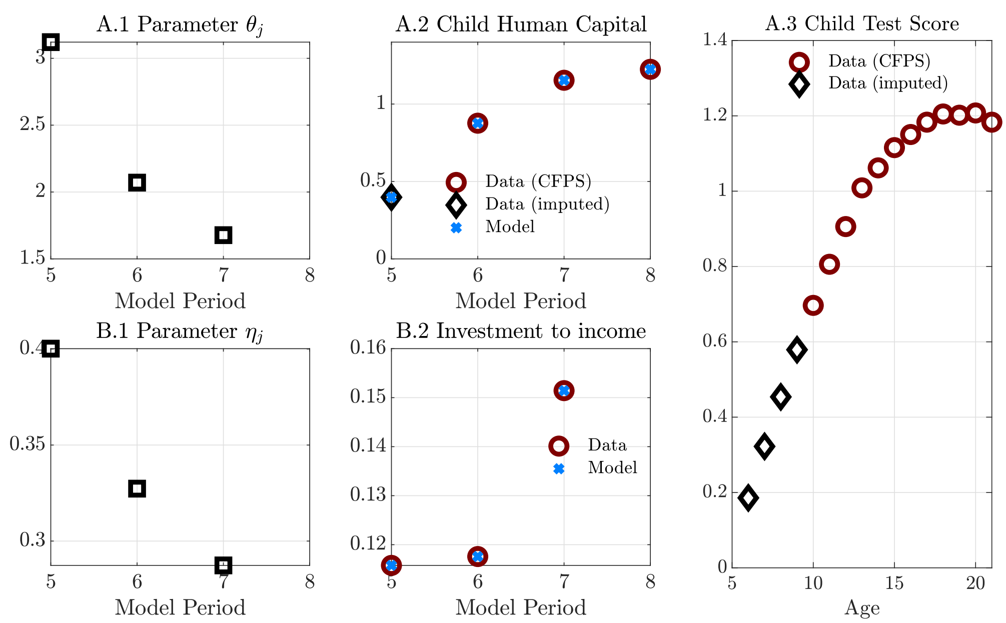

Figure 3: Age-dependent Parameters and Targeted Moments

Note: Panel (A.1) displays the estimates of θ by age j in the model. Panel (A.2) demonstrates how well the model moments on children's pre-college human capital by age j in the model, primarily used to discipline θ_j, align with their empirical counterparts. Panel (A.3) exhibits the data patterns on children's cognitive ability test scores by age, with the portion before age 10 imputed through extrapolation. Note that since one model period corresponds to 4 years in the data, we calculate the average cognitive ability scores of children aged 6-9, 10-13, 14-17, and 18-21, respectively, to compute the empirical moments used to discipline the model. Panel (B.1) presents the estimates of η by age j in the model. Panel (B.2) shows how well the model moments on the ratio of parental investment to labor income by age j in the model, primarily used to discipline η_j, match their empirical counterparts.

Given observations of both private and public investments and ability at an annual frequency, one could directly estimate the production function. However, since the CFPS provides skill and private investment measures only every four years without any information on public investments, we follow Abbott (2022) and use an indirect approach. The main identification comes from matching estimates of two misspecified translog child skill production function parameters with those estimated in the data. The first auxiliary model is

$$\log(i_j) = \alpha_0 + \alpha_1 \log(h_j) + \alpha_2 \log(y_j), \tag{20}$$

which describes how parental monetary investment depends on the current period's child cognitive skills and parental income.

The second model is

$$\log(h_{j+1}) = \beta_0 + \beta_1 \log(i_j) + \beta_2 \log(h_j) + \beta_3 \log(i_j) \log(h_j), \tag{21}$$

which captures how parental monetary investment, the current stock of child cognitive skills, and their interaction influence future child skills. For running both regressions specified in Equation (20) and (21) in the data, we control for both year and age effects. In the data, we observe parental monetary investment i_j for children aged 6-9 (model period 5), 10-13 (model period 6), and 14-17 (model period 7), and the child cognitive ability test scores h_{j+1} for ages 10-13 (model period 6), 14-17 (model period 7), and 18-21 (model period 8). We perceive child cognitive skills at ages 18-21 as the final outcome of pre-college human capital. For child skills before age 10, we infer them through extrapolation as discussed below.

Preferences We choose the discount rate $\beta = 0.89$ (which means the annualized discount factor is equal to 0.97) to target the standard 4% annual real interest rate. We discipline the altruism parameter ν along with other parameters that govern the child skill production function, as discussed below.

Child skill production function Our skill formation technology specified in Equation (5) incorporates three widely recognized features from the literature (e.g., Cunha and Heckman (2007), Cunha, Heckman, and Schennach (2010), and Heckman and Mosso (2014)). The first is multi-stage technology, which allows us to identify critical periods when parental investment can be more productive in producing child human capital. The second is self-productivity, which enables child skills produced at one stage to augment the skills attained at later stages. The third is dynamic complementarity, which allows skills produced at one stage to raise the productivity of investment at subsequent stages.

We choose the efficiency of child skill formation technology θ_j to match the age profile of child cognitive skills (Figure 3(A.2)). Since the CFPS data only allow us to observe cognitive ability test scores after children reach age 10, we extrapolate the age profile to infer average cognitive skills for ages 6 to 9 (Figure 3(A.3)), corresponding to the beginning of the model period 5.[33] We report the estimated results on θ_j in Figure 3(A.1).

We jointly calibrate the altruism parameter ν and the weights on parental investment $\eta_j, j = 5, 6, 7$, in the human capital formation function. The share of education expenditures in household income with respect to child age (Figure 3(B.2)) is sensitive to both parameters. However, the regression coefficient β_1 in Equation (21), which controls the effect of parental investment on next-period child skill, is only sensitive to the change in η_j. This feature allows us to separately identify the two parameters. We report the estimated results on η_j in Figure 3(B.1).

We choose the self-productivity parameter α, or the weight assigned to the current-period child skill as an input in the pre-college human capital production function, to match the standard deviation of log child cognitive skills at ages 18-21 ($j = 8$), which is considered as the final period of child pre-college human capital formation. In our model, variations in child final outcomes are driven by (1) variations in initial skill endowment (i.e., innate ability) and subsequent child skill stock and (2) variations in parental investment. In our model, an increase in child skill self-productivity leads to greater variations in child final human capital, allowing us to pin down α.

The dynamic complementarity parameter γ is set to match the regression coefficient β_3 in Equation (21) following Abbott (2022).[34] In cases where parental investment exhibits a stronger complementarity with the current-period child skill, the interaction term would exert a more negative impact. The elasticity of substitution between private and public investment, μ, is set to match the regression coefficient α_2 in Equation (20). Since public education investment, g_j, does not depend on household characteristics, a higher degree of complementarity leads to a smaller impact of household income, y, on private education expenditures, i. We obtain $\mu = 0.19$, indicating that governmental and parental monetary investments are substitutes, albeit far from perfect ones.[35]

[33] We have selected the initial stage to be ages 6-9, despite lack of cognitive ability score data in this range, because (1) we can still observe parental investments in children aged 6-9; and (2) The education competition intensity typically begins to increase in elementary school in China, starting at age 6.

[34] Our estimation result $\gamma = 0.42$ is also close to the one estimated in Abbott (2022), which equals 0.40, although we use different data sources and consider scenarios in different countries.

[35] This value suggests governmental and parental monetary investments are more than Cobb-Douglas. This finding is qualitatively consistent with the 0.59 estimation from Kotera and Seshadri (2017), which uses U.S. data and variations in public investment across school districts.

Table 3: Internally Estimated Parameters

Description	Parameter	Value	Targeted Moment	Data	Model
(A) Preference					
Discount factor	β	0.89	Real interest rate	0.04	0.04
Altruism	ν	0.37	Regression coefficient β_1	0.09	0.09
(B) Pre-college human capital formation					
Age-dependent efficiency of skill production, $j = 5, 6, 7$	θ_j	Figure 3(A.1)	Child cognitive skill age profile	Figure 3(A.2)	
Age-dependent weight on parental investment, $j = 5, 6, 7$	η_j	Figure 3(B.1)	Parental investment to income age profile	Figure 3(B.2)	
Weight on current child skill	α	0.64	SD of log child cog. skill (age 18-21)	0.27	0.27
Dynamic complementarity	γ	0.42	Regression coefficient β_3	-0.02	-0.02
Elasticity of substitution between i and g	μ	0.19	Regression coefficient α_2	0.58	0.58
(C) Intergenerational skill transmission					
Persistence of IG skill	ρ_h	0.07	IG correlation of cog. skill (age 10-13)	0.27	0.27
Avg. child skill endowment (college parent)	μ^{col}	0.32	Avg. rel. child cog. skill by parent edu (age 10-13)	1.13	1.13
Avg. child skill endowment (non-college parent)	μ^{ncol}	0.34	Avg. child cog. skill (age 6-9)	0.39	0.39
SD IG genetic shocks (college parent)	σ_h^{col}	0.11	SD log child cog. skill (college parent, age 10-13)	0.27	0.27
SD IG genetic shocks (non-college parent)	σ_h^{ncol}	0.12	SD log child cog. skill (non-college parent, age 10-13)	0.36	0.36
(D) Labor productivity shocks					
Persistence of labor productivity shocks	ρ_z	0.71	Estimated persistence of wage income	0.79	0.79
SD of labor productivity shocks	σ_z	0.21	Urban household wage income Gini coefficient	0.38	0.38
(E) Education and returns					
Conversion rate (pre-college to productive human capital)	λ	0.46	Regression coefficient α_1	1.14	1.14
College labor intensity (firm production)	ϕ	0.29	College labor income premium	1.72	1.72
College admission function shifter	ζ	-0.06	College-educated labor share of child generation	0.35	0.35

Intergenerational persistence of skills Since child human capital is only observed from age 10 onward, we use the intergenerational correlation of cognitive skills at ages 10-13 (model period 6) to identify the persistence of pre-college human capital, ρ_h. This persistence captures the influence of parental human capital on the child human capital endowment. The average and standard deviation of the human capital endowment are both dependent on parental education. We discipline the four parameters $\{\mu^{col}, \mu^{ncol}, \sigma_h^{col}, \sigma_h^{ncol}\}$ to target their counterparts in the CFPS data.

Labor income process We jointly calibrate ρ_z and σ_z to match the persistence of the labor income process and the Gini index of urban household labor income in China, respectively.[36]

Identifying the conversion parameter As discussed in Section 2.8, λ in our framework captures how pre-college human capital converts to productive human capital, which governs the relative strength of the competition incentive for parental investment, making it the most crucial parameter to our paper. There is no consensus in the existing literature on how to identify λ,[37] and none of the studies interpret it as a parameter that governs the conversion rate between the two types of human capital, and link it to the relative strength of the competition for college admissions driving parental investment.

We offer a novel identification strategy that leverages the non-monotonicity of parental investment with respect to child ability, a distinct empirical pattern observed in Chinese data, to identify λ.[38] More specifically, we use Equation (18) and *Insight 2* in Section 2.8 to demonstrate our identification strategy. This implies that to identify λ, we can investigate the dependence of parental investment on a child's cognitive abilities after

[36]The estimates on the persistence of the labor income process are drawn from Yu and Zhu (2013), which uses longitudinal data from the CHNS. The CHNS holds an advantage over the CFPS because of its longer panel feature. The 0.38 Gini index of urban household earnings is documented by Ding and He (2018) using the UHS data of the late 2000s.

[37]For example, in the quantitative macro literature, Abbott, Gallipoli, Meghir, and Violante (2020), Daruich (2022), and Abbott (2022) interpret λ as ability gradient. Abbott, Gallipoli, Meghir, and Violante (2020) and Daruich (2022) rely on a simple OLS regression of wages on the cognitive ability skills of adult workers using the National Longitudinal Survey of Youth (NLSY) data to directly estimate λ. Abbott (2022) internally calibrates λ to match the variance of the log of wage residuals. Lee and Seshadri (2019) simply assume that λ equals one. The labor literature usually focuses on dealing with issues on unobserved child endowment and endogeneity of observed inputs in the skill production function (see, for example, Todd and Wolpin (2003); Cunha and Heckman (2008); Del Boca, Flinn, and Wiswall (2014); Chan and Liu (2024)). As mentioned before, due to our data limitations, we do not observe individuals' childhood development stages (especially cognitive ability scores at around age 18) and labor market outcomes simultaneously, so a simple OLS (or IV) regression of wages on an ability measure cannot be used to estimate λ directly.

[38]In Figure A1, we report how parental monetary investments depend on child cognitive ability after controlling for parental income for both China and the United States, which look strikingly different.

taking parental income and college wage premium into consideration. In the data, we find a non-monotonic relationship between the parental investment-to-income ratio and child abilities, as reported in Figure 4. We use this information to distinguish between the "real" and "college competition" components of parental investments.

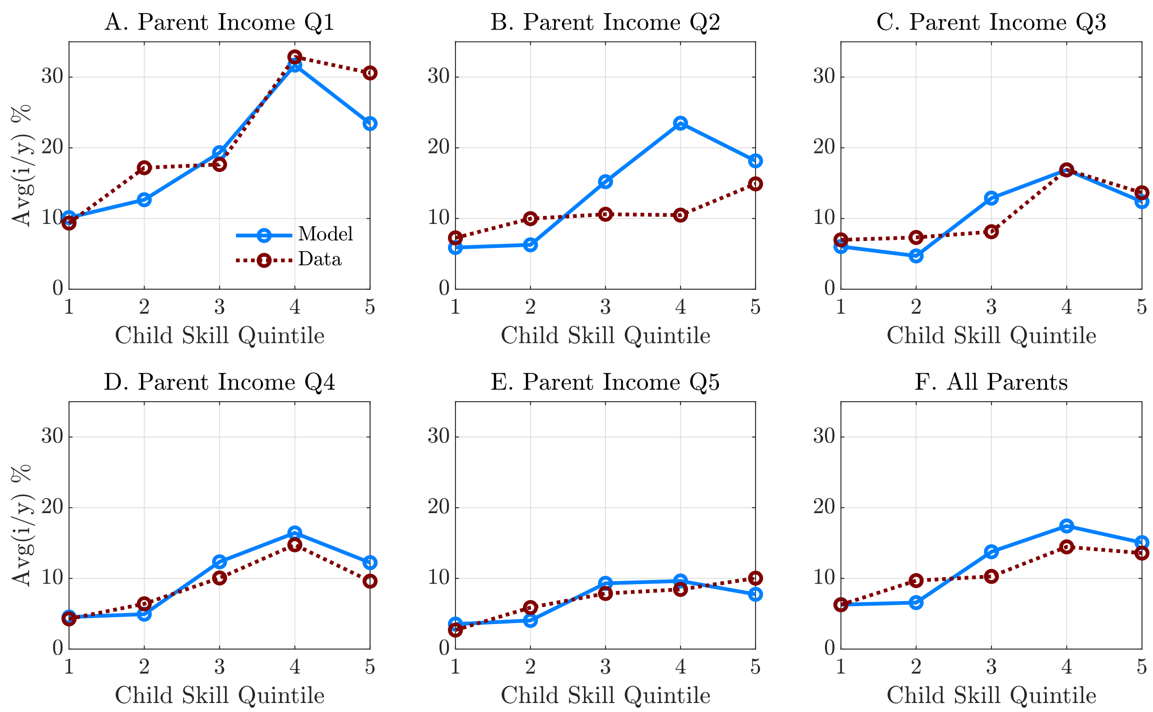

Figure 4: How parental investment depends on child skill after controlling parent income

Note: The child age group considered here is ages 10-13 (i.e., model period 6; patterns shown here also hold for other age groups of children). The horizontal axis represents the child cognitive skill quintile, while the vertical axis shows the corresponding average parental monetary investment to income ratio for each child skill quintile.

We discipline λ to match the regression coefficient α_1 in Equation (20), which captures how parental monetary investment i_j depends on child cognitive skills in the current period h_j after controlling for parental income y_j and year fixed effects. Simultaneously, we adjust the admission policy function shifter parameter ζ to align with the 35% share of college-educated labor in the children's generation as well as the college labor intensity parameter ϕ in the representative firm's production technology (Equation (3)) to match the college labor income premium of 1.72 found in our CFPS sample.[39] That is, we target

[39]Note that in the CFPS, there is no data available on the measurement of the intensive margin of labor supply, such as hours or weeks worked per year. Thus, we can only compute the average labor earnings for individuals with or without a college degree. As a result, we could only target college labor income premium instead of college wage premium.

$\frac{w^{col}\sum_{j=2}^{11}\int_{X_j} y_j^{col}(x_j)d\Psi_j}{w^{ncol}\sum_{j=1}^{10}\int_{X_j} y_j^{ncol}(x_j)d\Psi_j} = 1.72$ in the model where $y_j^s = z_j \exp(A_j^s) h_p^\lambda$. This implies that given the college labor income premium fixed, a higher λ also means a lower college wage premium $\frac{w^{col}}{w^{ncol}}$, thus a weaker competition incentive. Based on our calibration results, we find that λ equals 0.46. This means that pre-college human capital converts to productive human capital in a decreasing returns to scale manner. Consequently, the conversion rate diminishes as pre-college human capital increases, as shown in Figure A4.

The empirical patterns from Figure 4 reveals that, after controlling for parental income quintile, parental investment significantly increases when a child's skill reaches the fourth quintile in the distribution and then declines after the critical point. This is especially true for the panels representing the first, third, and fourth quintiles of parent income. We show that our model incorporating competition for college admissions with a well estimated value of λ rationalizes this fact very well. This can be attributed to the fourth quintile of skill being situated within the sensitive region of the college admission probability function, as illustrated in Figure 2(A). In this region, a small increase in pre-college human capital can substantially boost a child's admission probability, leading to a large private return on investment.

3.4 Model Performance

Our model successfully replicates several crucial non-target moments. The most important ones relate to how parental monetary investment depends on child skills after controlling for the parent income quintile, as shown in Figure 4. Note that to identify λ, we only target the regression coefficient α_1 in Equation (20). We present additional empirical moments not targeted in the calibration, along with their model-generated counterparts in Table A2 in Appendix C.1.

4 Impacts of the Competition Incentive on Parental Investment and Child Outcomes

In this section, we quantitatively investigate the role that the competition incentive plays in driving parental investment and the subsequent child outcomes, including the implications for sources of lifetime inequality and intergenerational mobility.

4.1 Shutting Down the Competition Incentive

We eliminate competition through three methods. In the first approach, we relax the college capacity for the child generation in our benchmark model such that everyone who wants to go to college can do so (the parent generation college capacity remains fixed at 7.5%, as in the benchmark). In the second approach, we make college attendance a purely random draw, independent of any individual heterogeneity including pre-college human capital.

In the third method, we raise the tuition parameter κ in Problem (10) to ensure that the proportion of parents who value sending their children to college more than not sending them aligns with the benchmark college capacity of 35%. This can be viewed as an alternative college admission system characterized by high tuition and fees. More specifically, we implement this alternative high-tuition college admission scheme as follows. The college education choice is now made endogenously by comparing two values:

$$v_8(s, a, h_p, z, h) = max\{v_8(s, a, h_p, z, h, col),\ v_8(s, a, h_p, z, h, ncol)\}, \tag{22}$$

Recall the value of agents whose children go to college $s_c = col$ is

$$v_8(s, a, h_p, z, h, col) = max\left\{u(c) + \beta\mathbb{E}_{z'}\left[v_9(s, a', h_p, z')\right] + \nu\beta\int v_2(0, h, z, col)\mu(z)\right\},$$

subject to

$$(1 + \tau_c)\,c + a' + \kappa = (1 + r)a + w^s y(h_p, A_t^s, z),\ c, a' \geq 0.$$

where tuition κ is raised such that the share of parents whose value of sending their children to college exceeds that of not sending them and is equal to the college capacity of the benchmark level, 35%. Consequently, κ now equals 1.15, increasing from the benchmark level of 0.17. This means the tuition costs paid by households in this high-tuition economy is 15% higher than the average household income.[40] For all other model parameters, we use the same ones as in the benchmark. Note that such a college admission process is closer to the one considered in quantitative Aiyagari-style life-cycle models with intergenerational linkages in the U.S. context (e.g., Abbott, Gallipoli, Meghir, and Violante (2020), Lee and Seshadri (2019), and Daruich (2022)).

[40] Note that in the benchmark economy, the sum of tuition costs paid by private households and college subsidies provided by the government equals 0.68, which is approximately 30% lower than the average household income. This implies that in this high-tuition economy, to balance the government budget as before, the linear consumption tax will need to be lowered.

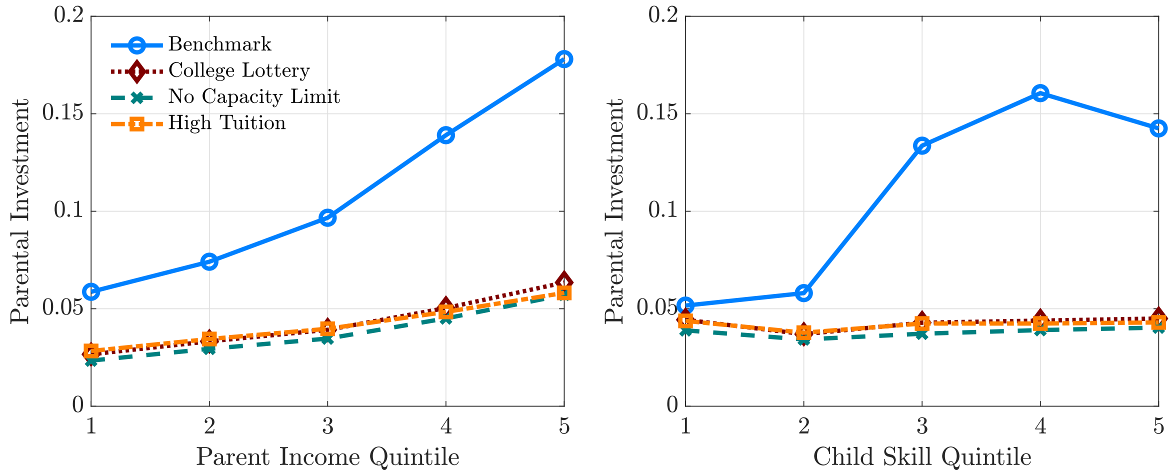

Figure 5: Parental Investment: Compare with No-Competition Scenarios

Note: The figure displays parental investment at $j = 6$, representing parental investment when children are between the ages of 10-13 (note that the average household income is normalized to be one). The blue-circle-solid line represents the benchmark case. The violet-diamond-dotted line represents the random draw case. The green-cross-dashed line represents the case of relaxing capacity constraints. The orange-cube-dashed line represents the high tuition case. More detailed results can be found in Table A4 in Appendix C.

We report patterns on how parental investment depends on child skill and parent income in Figure 5. We compare the three cases without the competition incentive (i.e., no-college-capacity constraint, lottery draw, and high tuition) with the benchmark. All three cases on eliminating competition exhibit strikingly similar patterns. This indicates that the competition incentive in the benchmark drives the observed investment patterns across parental income and child skills.

As displayed in Figure 5, without the competition incentive, investments for all parental income quintiles decline significantly, including those of low-income parents (bottom quintile). In contrast, there is minimal change in investments for children with low skills (bottom quintile) but a substantial decline for children with skills close to the sensitive region of the college admission function (the third to fifth quintiles). This finding implies that the competition incentive leads low-income parents to prioritize investing in their children, even at the expense of their consumption when their children's abilities are sufficiently high. Consequently, dampening the competition incentive through policies could result in considerable distributional welfare effects.

Decomposing Parental Investment Incentives Based on our approaches to eliminate the competition incentive, we can decompose parental monetary investment for each period ($j = 5, 6, 7$) into two components—one driven by the *standard incentive* that

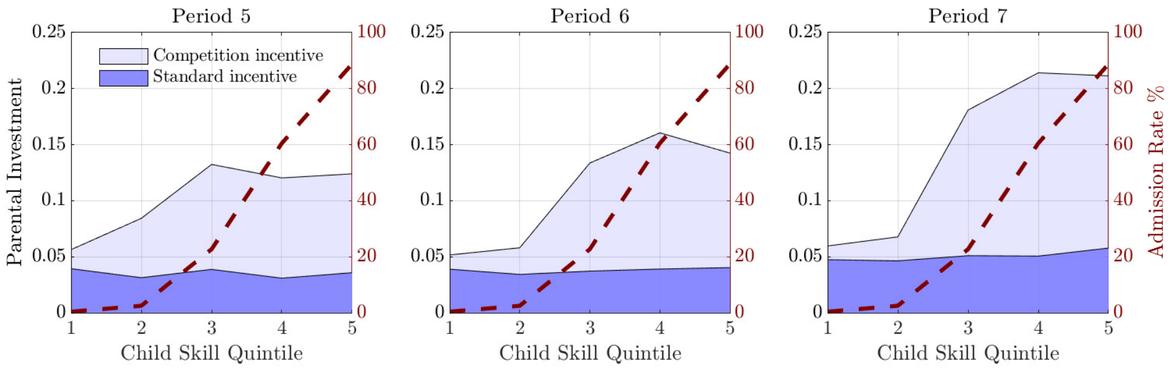

Figure 6: Decomposing Sources of Parental Investment

Note: The x-axis represents the quintiles of child pre-college human capital. The left y-axis represents the corresponding parental investment (note the average household income is normalized to be one). The right y-axis represents the corresponding college admission probability (at period 8) suggested by the model. Note that this is different from the estimated human-capital-based college admission probability function shown in Figure 2(A). We eliminate competition via relaxing college capacity constraint.

contributes to labor efficiency units and the other driven by the *competition incentive* aimed at securing college admission. We decompose the incentives by shutting down the competition incentive as discussed above. We report the results in Figure 6. In conjunction with the decomposition results on parental investment incentives, we also report how the college admission probability (plotted on the right y-axis) depends on child pre-college human capital quintiles, which are endogenously generated by the model. Overall, the competition incentive accounts for approximately 61-65% of parental investment for an average household, while the standard incentive makes up the remaining 35-39%, depending on how we eliminate the competition incentive.

This set of figures conveys three key messages. First, the point at which parental investment surges corresponds to the child skill level where the college admission probability also experiences a significant increase. Second, if parental investment is solely driven by the standard incentive (deep purple region), it is almost independent of child skill. Moreover, parental investment increases more as it gets closer to the college decision stage (i.e., from periods 5 to 7).[41]

The strength of competition incentive with respect to λ As discussed in Section 2.8 and 3.3, λ governs the conversion rate of pre-college human capital and productive human capital and thus the relative strength of the competition incentive for parental investment.

[41]This aligns with survey data from the China Institute for Educational Finance Research (CIEFR), which indicates that private education expenditures in urban China, on average, increase from 8.7k yuan at the elementary school stage, to 11.3k yuan at the middle school stage, and finally to 19.8k yuan at the high school stage.

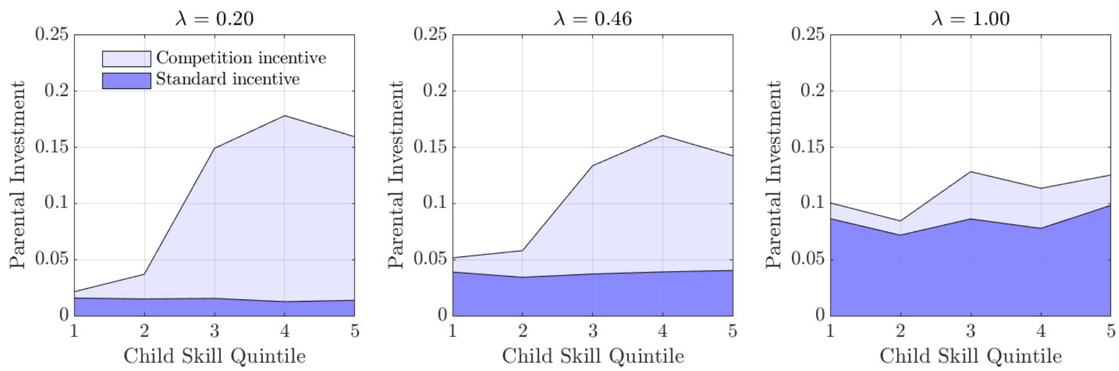

Figure 7: Sources of Parental Investment by λ

Note: This figure displays parental investment at $j = 6$, representing the parental investment when children reach the ages of 10-13. We eliminate competition via relaxing college capacity constraint in this figure.

Next, we consider varying the value of λ. We choose three levels for λ: a low $\lambda = 0.20$, the baseline $\lambda = 0.46$, and a high $\lambda = 1.00$.[42] For each level of λ, we recalibrate all the model parameters as we do in Section 3.3 except how parental monetary investment depends on child skills after controlling for parental income (regression coefficient α_1 in Equation (20)) to make the three cases comparable.[43] From Equation (2), the larger the λ, the higher the conversion rate from pre-college human capital to productive human capital.[44] Consequently, as shown in Figure 7, the competition incentive becomes weaker, as a smaller fraction of parental investment is driven by the competition incentive.

4.2 Child Outcomes and Lifetime Inequality

Child Outcomes Considering the significant role that the competition incentive plays in driving parental investment and its heterogeneous impacts based on family characteristics, it should also affect child outcomes differently depending on the child's background in terms of parental income and her own ability. We analyze the impact of the competition incentive on the expected pre-college human capital of children at the age of 18, taking into account parental background and child innate ability, following an approach similar to decomposing parental investment incentives. As shown in Figure 8, the blue area is

[42]We aim to select a relatively low λ without setting it to zero, as this would result in parental investments being driven solely by the competition incentive, leaving public investment g_j with no role. We choose our low λ to be 0.2, which is close to the lowest possible ability gradient estimated by Abbott, Gallipoli, Meghir, and Violante (2020) for female workers who have not completed high school education. For our high λ, we select it to be one, consistent with the value used in Lee and Seshadri (2019).

[43]See Table A3 in Appendix B for more details on recalibration.

[44]In our model, the entire distribution of pre-college human capital ranges approximately from 0.5 to 2 in which the conversion rate increases with a higher λ. See Figure A5 in Appendix C for more details.

driven by the competition incentive.[45]

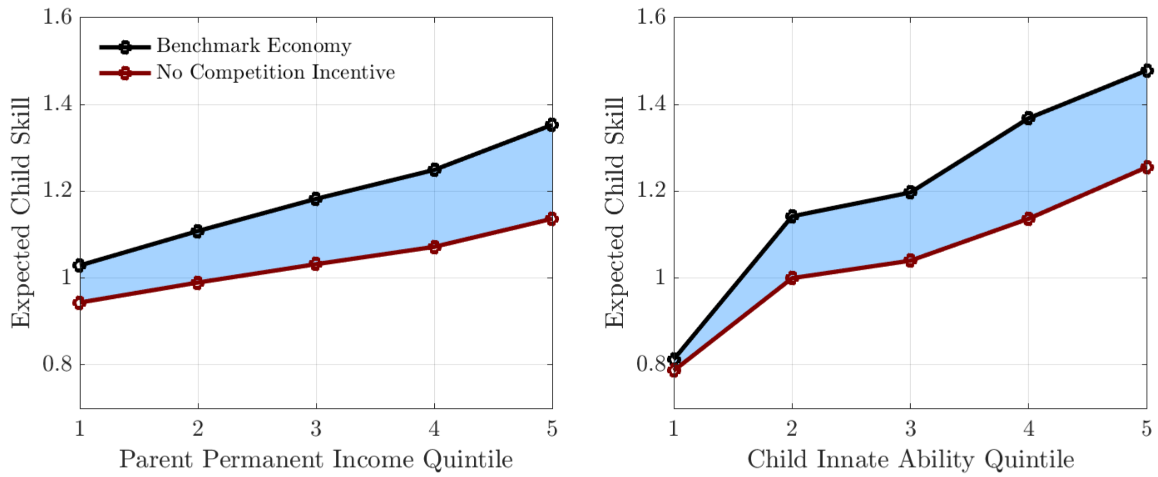

Figure 8: Impact of the Competition Incentive on Child Human Capital

Note: This figure plots how the expected pre-college human capital of a child at age 18 depends on the parent's income quintile and the child's initial human capital endowment quintile. The parent income concept we used here is permanent income, which is defined as $w^s \mathbb{E}_z y(h_p, A^s, z)$.

We offer two main takeaways here. First, regardless of the presence of competition, a child's expected pre-college human capital relies more on their innate abilities than their parental background. More importantly, removing competition has the most significant impact on the pre-college human capital of children with relatively high innate abilities. This is consistent with the parental investment incentive decomposition results found in Figure 6. Concerning parental background, the absence of competition has a greater effect on children with relatively high parental income; however, this reduction is not as substantial as the decrease observed in children with sufficiently high innate abilities.

Sources of Lifetime Inequality and Mobility In the macroeconomics literature on inequality and mobility, a key question to answer is whether lifetime inequality is primarily due to differences in initial conditions determined early in life or to differences in luck experienced over the working lifetime. In the seminal work by Huggett, Ventura, and Yaron (2011), the authors find that differences in initial conditions account for a larger portion of the variation in lifetime earnings. This conclusion remains consistent after endogenizing initial conditions, as shown in Lee and Seshadri (2019) and Daruich

[45] Note that the competition incentive does not play as significant a role in driving child pre-college human capital as it does in driving parental investment. The primary reasons are that besides parental investment, both public investment and early-stage child human capital are also crucial inputs in producing child human capital, as suggested by the child skill production function (Equation (5)).

Table 4: Sources of Lifetime Inequality

	Benchmark	**High tuition**
Inequality		
Variance log Lifetime Earnings	0.14	0.11
% expl. by initial labor market conditions	51%	39%
% expl. by child innate ability	28%	11%
% expl. by parenting	15%	4%
% expl. by college attendance uncertainty	8%	-
% expl. by college tuition affordability	-	24%
% expl. by adult income shocks	49%	61%
Correlation		
Corr(h_5, h_8)	0.83	0.87
Corr$(h_5, s_c = col)$	0.61	0.02
Corr$(h_5, \mathrm{E}[\mathrm{LTE}])$	0.75	0.41
Corr$(h_8, s_c = col)$	0.86	0.20
Corr$(h_8, \mathrm{E}[\mathrm{LTE}])$	0.96	0.61
Intergenerational Persistence		
Correlation coefficient		
Pre-college human capital	0.37	0.31
Education	0.23	0.26
Lifetime earnings	0.40	0.46

(2022). We attempt to answer this question as well, by taking into account the non-trivial competition incentive for parental investment in the Chinese economy. Furthermore, we aim to explore, through the lens of our model, what factors contribute to differences in initial conditions—whether it is due to *nature* (i.e., child innate ability) or *nurture* (i.e., parenting)—thereby enhancing our understanding of the sources of lifetime inequality and intergenerational mobility.

We decompose the variance of lifetime earnings into components attributed to initial conditions and adult income shocks in both the benchmark model and the high-tuition scenario, which represents another widely used college admission scheme in the world where the competition incentive—in the form of a parental monetary investment to build up a child's pre-college human capital to secure college admission—is absent. We report results in Table 4. An agent's career begins with a certain level of pre-college human capital, resulting from investments by parents and the government. In the benchmark case, initial labor market conditions account for approximately 51% of variations in lifetime earnings, with 28% attributed to children's innate ability (i.e., initial human capital endowment), 15% to parenting, and 8% to college attendance uncertainty driven by our estimated pre-college human-capital-based college admission policy in Figure 2(A). Adult income shocks account for the remaining 49% after the individual becomes independent. Our results align with Huggett, Ventura, and Yaron (2011), Lee and Seshadri (2019), and Daruich and Kozlowski (2020) despite differing model choices.

Interestingly, while the competition incentive drives significant parental investment in children with marginal abilities, *nurture* plays a less crucial role in lifetime earnings inequality than *nature*. This is attributed to the uneven distribution of parental investment in pre-college human capital. Children with a large human capital endowment receive disproportionately more investment from parents, as evidenced by the strong positive correlation between a child's human capital endowment (h_5) and pre-college human capital at age 18 (h_8). This also indicates that the substantial investment driven by the college competition incentive does not significantly alter the ranking of h_8 relative to h_5,[46] and thus has little impact on college admission outcomes. Consequently, the competition for college admissions does not result in severe talent misallocation issues. However, it does create "wasteful" investments due to the relatively low conversion rate from pre-college human capital to labor efficiency units, while only minimally affecting college admission outcomes. In general, the human-capital-based college admission scheme makes h_5 a strong predictor of college education outcomes and expected lifetime earnings, suggested by the high correlation between h_5 and child education outcomes ($s_c \in \{col, ncol\}$) as well as between h_5 and children's expected lifetime earnings.

In the high-tuition scenario, where the competition incentive is absent, only 39% of the variations in lifetime earnings are explained by initial conditions, with college tuition affordability accounting for the largest part. Consequently, child innate ability h_5 (and thus pre-college human capital upon entering the labor market h_8) have smaller predictive power in college education outcomes and expected lifetime earnings.[47] These results also explain why, in terms of intergenerational mobility, the alternative high-tuition college admission scheme results in higher intergenerational correlations for education and lifetime earnings compared to the benchmark scheme. Since the high-tuition scheme places greater importance on the affordability of tuition and fees, children with college-educated, high-income parents are more likely to attend college and achieve high lifetime earnings.

[46] Note that parental investment is the only factor that affects the correlation between h_5 and h_8. Without the intervention of parental investments, the correlation is simply one.

[47] The correlation between h_5 and h_8 is also high in the high-tuition case (0.87) because parental investments do not heavily depend on child ability, which implies a child with high innate ability is more likely to remain highly skilled at age 18.

A comparison of the results on sources of lifetime inequality under the benchmark ability-based college admission scheme featuring a strong competitive incentive and an alternative high-tuition college admission system further explains why studies modeling the U.S. college admission system emphasize the importance of parental income for child achievement and educational attainment, as college tuition affordability plays the biggest role in driving lifetime inequality. In contrast, our benchmark scenario, where the competition incentive is prevalent, places child ability at the center of the analysis on child development and corresponding policy guidance due to the extremely high complementarity between child innate ability and parental investment. This further suggests that policymakers may need to pay greater attention to children with relatively low abilities (in addition to those from low-income households), who may consistently face disadvantages due to receiving limited private investment.

5 Policy

Previous sections demonstrate that a significant portion of parental investment toward children is due to the competition incentive, with the portion particularly large for children with abilities close to the college admission threshold, at the expense of parents' consumption. This suggests that regulating competition could potentially improve overall welfare. In this section, we quantitatively evaluate the aggregate, welfare, and distributional effects of regulating parental investment competition in college admissions from both positive and normative perspectives. We begin by analyzing impacts of China's 2021 private tutoring ban policy on human capital accumulation, lifetime income, welfare, inequality, and mobility in both the short and long term. We then explore the possibility of achieving better results through a private education investment tax.

5.1 Short-Run Impacts of the Private Tutoring Ban

In 2021, China banned for-profit tutoring in core school subjects—aimed at passing exams—to alleviate financial pressures on families arising from intense competition for college entrance examinations.[48] We implement this policy in our model by setting a cap on parental investment beyond which investment would not be allowed. The cap, $\bar{i}$, is

[48] On July 24, 2021, the Chinese government officially issued the Guidelines for Further Easing the Burden of Excessive Homework and Off-campus Tutoring for Students in Compulsory Education, aiming to foster student well-being, enhance educational quality, reduce financial strain on parents, and establish law-based governance in the education sector. The reforms focus on core subjects in compulsory education, encompassing grades K-9 and catering to students aged 6-15 years.

Table 5: Short-Run Aggregate Outcomes for Private Tutoring Ban

	Benchmark	Ban
(i) **Aggregates**		
Parent investment	0.12	-39.18%
Parent consumption	0.92	+1.55%
Child pre-college human capital	1.20	-5.78%
Child expected lifetime earnings	1.27	-2.83%
(ii) **Welfare**		
Consumption equivalent	-	+0.49%
(iii) **Inequality**		
Var log lifetime earnings	0.14	-8.10%
(iv) **Intergenerational Persistence**		
Pre-college human capital	0.37	0.26
Education	0.23	0.06
Lifetime earnings	0.40	0.23

Note: Consumption equivalence here is calculated based on the parent generation under the veil of ignorance. Inequality refers to the variance of log-expected lifetime earnings for the child generation. Intergenerational persistence is measured in terms of correlation coefficients.

chosen such that the share of households with investment $i > \bar{i}$ equals 0.52, as our data indicate that 52% of parents invest in private tutoring in addition to other categories of monetary investment.[49] We also adjust the college admission shifter parameter, ζ, and linear consumption tax, τ_c, to match the 35% college capacity and balance the government budget. To evaluate the short-term effects, we impose the private investment cap and conduct a one-generation-ahead transition without changing prices.

Pre-college human capital losses and welfare With the investment cap $\bar{i}$ in place, as shown in Table 5, parental investment declines by around 39% on average. This leads to an approximate 2% increase in parents' consumption and a 6% decline in children's pre-college human capital (at the age of entering college). Consequently, children's expected lifetime earnings decrease by an average of 3%. Welfare, measured in terms of the consumption-equivalent utility of the parent generation, is affected by the private

[49]This number is based on our computation using the 2018 wave of the CHIP. The rationale behind structuring private tutoring expenditures in this manner is as follows. Parents prioritize allocating funds toward supplying in-school necessities including stationery, books, boarding, and transportation before investing in supplementary private tutoring services.

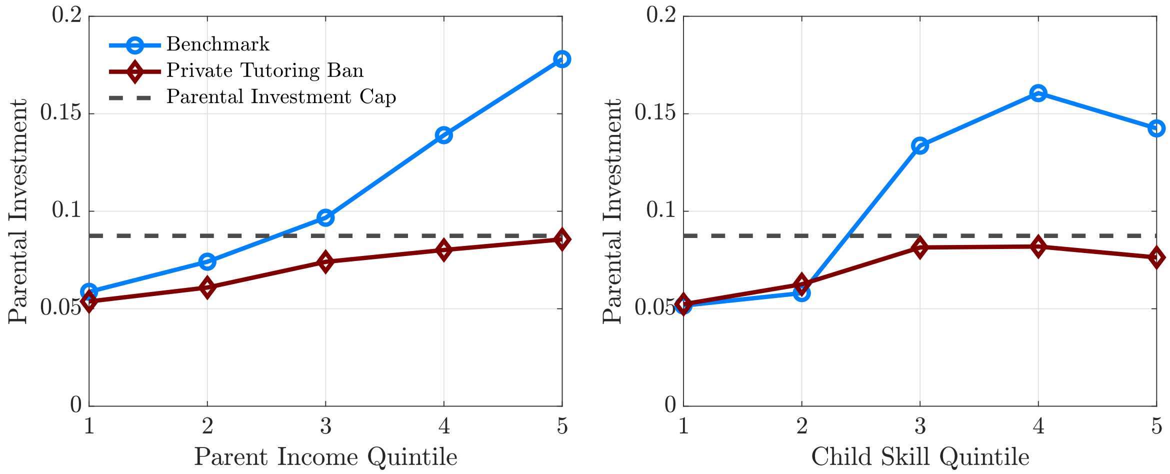

Figure 9: Parental Investment Before and After the Private Tutoring Ban

Note: This figure displays parental investment at $j = 6$ (note the average household income is normalized to be one). The blue-circle-solid line represents the benchmark case. The violet-diamond-dotted line represents case of the private tutoring ban. The parental investment cap, represented by the black dashed line, is a threshold above which the expenditures spent represent private tutoring spending. Under a private tutoring ban, investment above this cap is not allowed.

tutoring ban in two ways. On the one hand, reduced spending on children's education can increase parents' consumption and welfare (also working through improved investment efficiency for those who invest substantially due to the competition incentive). On the other hand, a decline in children's expected lifetime utility due to human capital losses reduces parents' welfare since they care about their children's well-being for altruistic reasons. Overall, parents' average welfare increases by 0.5%.

Inequality and mobility As shown in Figure 9, private tutoring ban reduces private investment disproportionately from high-income families. More importantly, private investment disproportionately decreases for families with children's pre-college human capital on the margin. These results align with those observed in Figure 5 when the competition incentive is eliminated. Consequently, lifetime income inequality and intergenerational persistence both decline, as shown in Block (iii) and (iv) of Table 5.

Distributional effects We examine the distributional effects across heterogeneous families (indexed by parent income and child innate ability) on the welfare of the parent generation and child outcomes. The results are summarized in Table 6. Regarding the distributional welfare effects, families with low parental income (first quantile) and high child innate ability (third and fourth quantiles) benefit most from the private tutoring ban, whereas families with high parental income (fourth quantile) and marginal child innate ability (third quantile) lose the most. The competition incentive, along with altruism, leads low-income parents to prioritize investing in their children, even at the expense of their consumption, when their children's abilities are sufficiently high (i.e., third quantile or above), as suggested by the Panel (A) of Figure 4. When high-income parents are prohibited from investing further to compete with high-ability children from low-income families, their children may not secure college admissions as they did before the regulation, resulting in the most significant reduction in their welfare. However, low-income parents with high-ability children no longer need to sacrifice their consumption as they did before, leading to the most substantial increase in their welfare.[50]

Regarding child outcomes, in terms of child expected lifetime income, children from low-income families (first quantile) with lower abilities (first and second quantiles) benefit the most from the private tutoring ban, whereas children from high-income families (fourth quantile) with higher abilities (third and fourth quantiles) experience the greatest losses. These outcomes are influenced by two main factors: college attendance and child human capital before entering the labor market. Analyzing the distributional effects on these factors, Panel (C) of Table 6 indicates that the ban significantly enhances college enrollment opportunities for high-ability children from low-income families through preventing affluent parents from securing college admissions of their own children via extra investments. Panel (D) of Table 6 shows a general decline in pre-college human capital for most children but an increase for children with low abilities from low-income families. This outcome is due to a leftward shift in the pre-college human capital distribution, leading parents of low-ability children to anticipate a higher likelihood of college admission and subsequently increase their investments.

Unintended Consequences To address the potential unintended consequence of the ban driving the private tutoring sector underground and enabling only wealthy parents to hire tutors, we double the price of parental investment for any amount exceeding the cap. This effectively mimics a 200% proportional tax on private tutoring expenses, leaving the tax revenue unused. It is worth noting that the private-tutoring-ban scenario simulates an infinite tax on such expenses. Consequently, it is not surprising that the qualitative results persist even after private tutoring goes underground, albeit with reduced magnitudes. The corresponding results can be found in Appendix C.6.1.

[50] We find highly similar patterns when considering parent human capital or parent wealth instead of parent income.

Table 6: Short-Run Distributional Effects for Private Tutoring Ban

(A) Welfare		Child Innate Ability			
		Q1	Q2	Q3	Q4
Parent Income	Q1	+0.44%	+0.88%	+1.71%	+1.91%
	Q2	+0.31%	+0.82%	+1.60%	+1.77%
	Q3	+0.31%	+0.71%	+1.31%	+1.31%
	Q4	+0.13%	+0.50%	-0.39%	+0.63%
(B) Child Income		**Child Innate Ability**			
		Q1	Q2	Q3	Q4
Parent Income	Q1	+1.84%	+3.13%	+3.33%	+3.08%
	Q2	+0.13%	+0.16%	-2.05%	+0.54%
	Q3	-0.36%	-4.10%	-5.02%	-3.95%
	Q4	-4.77%	-8.01%	-16.38%	-7.96%
(C) College Share		**Child Innate Ability**			
		Q1	Q2	Q3	Q4
Parent Income	Q1	+2.72%	+6.26%	+12.52%	+14.71%
	Q2	+0.54%	+2.85%	+3.43%	+9.66%
	Q3	+0.36%	-3.10%	-2.40%	-0.25%
	Q4	-3.70%	-9.41%	-25.34%	-8.96%
(D) Child Skill		**Child Innate Ability**			
		Q1	Q2	Q3	Q4
Parent Income	Q1	+1.35%	+1.13%	-2.68%	-3.67%
	Q2	-0.21%	-2.11%	-6.82%	-5.30%
	Q3	-1.10%	-6.17%	-8.87%	-8.26%
	Q4	-6.88%	-9.61%	-16.38%	-11.06%

Note: Each cell in the panels represents the percent change in a specific variable following the private tutoring ban for a combination of child pre-college human capital endowment quartiles and parent labor income quartiles. Panel (A) corresponds to the consumption-equivalent welfare of parents, Panel (B) to the child's expected lifetime income, Panel (C) to the college attendance probability for children, and Panel (D) to the pre-college human capital at age 18 for children.

5.2 Long-Run Impacts of the Private Tutoring Ban and the Optimal Private Education Investment Tax

In this section, we first assess the long-run impacts of the private tutoring ban. We then explore whether there is a policy that mitigates the distortion caused by the competition incentive without negatively affecting future generations. Although various policy instruments are available, we focus on a linear tax imposed on private tutoring expenditures.

We consider two methods for utilizing private tutoring tax revenue. In the first case, we evenly distribute the revenue among adult households as a lump-sum transfer. In the second case, we aim to enhance investment for children, especially those with low abilities, who receive minimal private investment, as analyzed in Section 4, by combining a linear private tutoring tax with a linear subsidy on pre-college public expenditures, funded solely by private tutoring tax revenue. We proceed to study the optimal private tutoring tax problem for both cases.

Since our benchmark model used for calibration is a non-stationary economy due to the significant gap in the college labor share between parent and child generations, we must first solve for a stationary equilibrium before evaluating the long-run policy impacts. In the stationary equilibrium, the college capacity (and therefore the college share) for all generations equals the level of the child generation in the benchmark (i.e., 35%). The considerable increase in the college labor share, jumping from 7.5% to 35% as a result of the large-scale college expansion, would cause the college wage premium $\frac{w^{col}}{w^{ncol}}$ to decline drastically if the demand side remains unchanged. Consequently, we adjust the college labor intensity parameter ϕ in firm production technology (Equation (3)) to ensure that the college wage premium $\frac{w^{col}}{w^{ncol}}$ is maintained at the benchmark level. This allows the relative strength of the competition incentive to remain the same as at the benchmark level. Simultaneously, we modify the college admission shifter parameter ζ and consumption linear tax rate τ_c to match the 35% college capacity and balance the government budget.[51] We then impose the cap $\bar{i}$ on private parental investment to simulate the policy of a private tutoring ban and solve for the new steady state.

5.2.1 Long-Run impacts of a private tutoring ban

We report the results in Column (ii) of Table 7. The private tutoring ban restricts private monetary investment from parents, causing investment to decline and pre-college human capital to deteriorate. Consequently, lifetime earnings decrease. Although people now spend less money on investing in human capital of their offspring, which has a positive impact on their consumption, the policy generates long-run human capital losses and thus earnings losses, eventually resulting in very little change in consumption. The key mechanism is that investing less in a child today not only reduces that child's human capital and income but also creates a less productive parent for the following generations. The consumption-equivalent welfare declines by around 0.6% on average. This contrasts with the short-run effects of a private tutoring ban on welfare, in which it increases by

[51] This means that when solving for the initial steady state in the long run, we need to search for five parameters/prices: $w^{col}, w^{ncol}, \phi, \tau_c, \zeta$.

around 0.5%.[52]

5.2.2 Private Education Investment Tax

We now investigate the effects of a private tutoring tax in our model. We denote the linear tax imposed on private tutoring expenditures by τ_i. We consider two cases depending on how private tutoring tax revenue is utilized.

For the case of a lump-sum transfer, the household budget constraints become

$$(1+\tau_c)\,c + a' + i + \tau_i \mathbb{1}_{i>\bar{i}}(i-\bar{i}) = (1+r)a + w^s y(h_p, A_j^s, z) + D, \tag{23}$$

where lump-sum transfer D is determined by the following financing constraint:

$$\sum_{j=1}^{15} \int_{X_j} D d\Psi_j = \sum_{j=5}^{7} \int_{X_j} \tau_i \mathbb{1}_{i>\bar{i}}(i-\bar{i}) d\Psi_j. \tag{24}$$

To capture the program that uses private tutoring tax revenue to subsidize pre-college public investment, we add one term to the household budget constraints:

$$(1+\tau_c)\,c + a' + i + \tau_i \mathbb{1}_{i>\bar{i}}(i-\bar{i}) = (1+r)a + w^s y(h_p, A_j^s, z), \tag{25}$$

The next period's pre-college human capital of children becomes

$$h' = \theta_j \left[\alpha h^\gamma + (1-\alpha)\, I_j^\gamma\right]^{\frac{1}{\gamma}}, I_j = \left[\eta_j i^\mu + (1-\eta_j)\left((1+\tau_g) g_j\right)^\mu\right]^{\frac{1}{\mu}}, \tag{26}$$

where the linear subsidy imposed on pre-college public investment τ_g is determined by:

$$\sum_{j=5}^{7} \int_{X_j} \tau_g g_j d\Psi_j = \sum_{j=5}^{7} \int_{X_j} \tau_i \mathbb{1}_{i>\bar{i}}(i-\bar{i}) d\Psi_j. \tag{27}$$

Starting with the initial steady state defined in Section 2.7, we solve the optimal private tutoring tax problem to maximize the long-run ex ante lifetime utility of newborns by choosing τ_i while satisfying the financing constraint, i.e., Equation (27) (or Equation (24)), through τ_g (or D), as well as the government budget constraint, the college capacity constraint, and market clearing conditions.

We report the results on the optimal tax in Column (iii) of Table 7. We find the optimal

[52] These welfare numbers appear relatively small for two primary reasons. First, there are opposing forces affecting welfare, as explained above. Second, we measure welfare through lifetime consumption equivalents, and parents invest in their children's pre-college human capital for only three model periods, or 12 years, which is a relatively short duration compared to a lifetime.

Table 7: Long-Run Policy Impacts

	(i) Initial SS	(ii) Ban	(iii) Optimal Tax transfer	(iii) Optimal Tax g_j subsidy
(i) Aggregates				
Parental investment	0.13	-40.56%	-8.89%	-8.90%
Effective investment	0.18	-10.60%	-3.71%	-1.26%
Pre-college human capital	1.22	-6.02%	-2.20%	-0.69%
Lifetime earnings	1.03	-1.55%	-0.37%	-0.09%
Consumption	0.91	-0.01%	-0.02%	-0.09%
Total output	0.83	-3.25%	-1.08%	-0.28%
(ii) Welfare				
Consumption equivalent	-	-0.58%	+0.07%	+0.21%
(iii) Inequality				
Var log lifetime earnings	0.14	-10.61%	-3.66%	-4.86%
(iv) Intergenerational Persistence				
Pre-college human capital	0.38	0.20	0.33	0.32
Education	0.38	0.08	0.33	0.32
Lifetime earnings	0.42	0.15	0.37	0.36

Note: Long-run impacts refer to looking at outcomes in the new long-run steady state with markets cleared, and the college capacity constraint and the government budget satisfied. Consumption equivalence is calculated based on ex ante newborns under the veil of ignorance. Inequality refers to the variance of log-expected lifetime earnings for the stationary distribution of households. Intergenerational persistence is measured in terms of correlation coefficients.

tax rates to be 23% for the lump-sum transfer case and 30% for the public investment subsidy case. Compared to a private tutoring ban, implementing a private tutoring tax significantly improves outcomes. The average pre-college human capital and lifetime earnings only slightly decline, primarily as a result of a much smaller reduction in private parental investment (approximately 9% in the private tutoring tax case versus 41% in the private tutoring ban). The welfare also increases, irrespective of how private tutoring tax revenue is used. Compared to the case of a ban on private tutoring, which leads to approximately 0.6% welfare reduction in the long run, a private tutoring tax balances dampening the competition incentive with minimizing human capital losses.

When we compare the lump-sum transfer case with the public investment subsidy case, it is evident that the usage of tax revenue matters a lot, with public investment

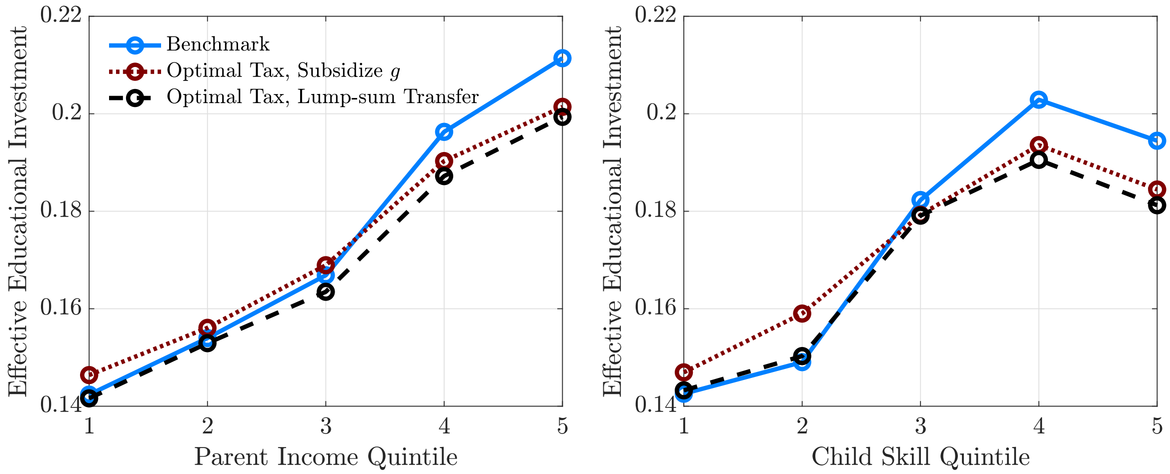

Figure 10: Effective Investment Patterns under Different Policy Scenarios

Note: This figure displays effective investment I at $j = 6$ (note the average household income is normalized to be one). Note that $I_j = \left[\eta_j i^{\mu} + (1-\eta_j)\, g_j^{\mu}\right]^{\frac{1}{\mu}}, j = 5, 6, 7$. The blue-circle-solid line represents the benchmark case. The violet-circle-dotted line represents the case of optimal tax with subsidizing g. The black-circle-dashed line represents the case of optimal tax with lump-sum transfer.

subsidies leading to larger welfare gains. The primary reason is that although private investment declines by almost the same amount in both cases, public investment subsidies result in a much smaller decline in effective investment compared to the lump-sum transfer case (1% vs. 4%). Note that effective investment in children's pre-college human capital is measured by aggregating private investment from parents and public investment from the government using a CES aggregator, as specified in Equation (5).

Regarding inequality and intergenerational persistence, the private tutoring ban prevents high-income parents from investing in their children's pre-college human capital to a large extent. Therefore, intergenerational persistence is closer to the correlation between parents' pre-college human capital and children's innate ability. That is why a private tutoring ban reduces lifetime inequality and intergenerational persistence much more than an optimal tax does. However, reducing inequality and intergenerational persistence to a larger extent does not necessarily mean that a policy is optimal, as it may cause significant human capital losses, particularly for children with high abilities. Moreover, as shown in Figure 10, the optimal private tutoring tax, combined with public investment subsidies, considerably improves the situation for children with disadvantaged backgrounds. Effective investment increases significantly for children with relatively low abilities and low parental income. As a result, lifetime income inequality declines by around 5%, and intergenerational persistence in lifetime income declines by around 14%.

6 Robustness

In this section, we analyze the robustness of our main findings on (1) how our key policy insight—taxing private education investment can be welfare-improving—is affected by the relative strength of the competition incentive, and (2) whether there exist alternative mechanisms that generate non-monotonicity in parental investment with respect to child ability, and whether these mechanisms result in differing policy implications.

We start by examining how the policy experiment results, discussed in Section 5, are influenced by altering the value of the conversion parameter λ and by adopting an alternative high-tuition college admission scheme where the competition incentive is absent. We then extend our benchmark model to allow for education-specific λ values, as in Abbott, Gallipoli, Meghir, and Violante (2020) and Daruich (2022), and perform the same exercises as in Section 4 (eliminating the competition incentive) and 5 (evaluating the impacts of private education investment tax). This is to address the concern that the complementarity of college education and child ability may also contribute to the observed non-monotonic parental investment patterns with respect to child ability, even in the absence of competition.

6.1 Private Education Investment Tax and Welfare Consequences by λ

As shown in Figure 7, the smaller the λ, the stronger the competition incentive, resulting in a larger fraction of parental investment being driven by the competition incentive. In this exercise, we compute the optimal private tutoring tax rates for various levels of λ with the private tutoring tax revenue used to subsidize pre-college public investment only. For each level, we recalibrate our model, compute a GE stationary equilibrium, and find the corresponding optimal tax rate. The results are reported in Table 8, with numbers representing changes relative to the initial steady state for each λ. The optimal tax rate increases as λ decreases, indicating a greater need for government regulation of parental investment competition. When λ is low, parents primarily invest in children's pre-college human capital to enhance their ability to perform well in exams and gain college admission. This necessitates more significant regulation, resulting in an optimal private education investment tax rate as high as 145% and larger welfare gains compared to the benchmark case (i.e., $\lambda = 0.46$). On the other hand, when λ equals one, pre-college human capital fully converts into productive human capital, and the competition incentive for parental investment is extremely weak, almost nonexistent. This reduces the need for government intervention, as evidenced by the 0% optimal tax rate.

Table 8: Long-Run Impacts of Optimal Education Tax under Various λ

	(i) $\lambda = 0.20$	(ii) $\lambda = 0.46$	(iii) $\lambda = 1.00$	(iv) $\lambda^{col} = 0.79$ $\lambda^{ncol} = 0.51$
Optimal Tax Rate	145%	30%	0%	15%
(i) Aggregates				
Private investment	-34.65%	-10.61%	-	-5.38%
Effective investment	-2.88%	-1.64%	-	-0.26%
Pre-college human capital	-2.01%	-0.91%	-	-0.10%
Lifetime earnings	-0.24%	-0.25%	-	-0.17%
Consumption	+1.18%	+0.19%	-	+0.04%
Total output	-0.48%	-0.49%	-	-0.36%
(ii) Welfare				
Consumption equivalent	+1.00%	+0.21%	-	+0.11%
(iii) Inequality				
Var log lifetime earnings	-6.27%	-4.86%	-	-1.67%
(iv) Intergenerational Persistence				
Pre-college human capital	-44.42%	-15.11%	-	-7.96%
Education	-55.80%	-17.46%	-	-8.98%
Lifetime earnings	-52.89%	-15.81%	-	-7.82%

Note: Column (ii) with $\lambda = 0.46$ represents the benchmark level. For Column (i) and (iii), we recalibrate all the parameters targeting the same set of moments as we do in Section 3.3 except the regression coefficient α_1 in Equation (20). For Column (iv), we explain our calibration strategy in Section 6.3.

6.2 Policy Impacts under an Alternative College Admission Scheme

In this exercise, we solve the long-run optimal private tutoring tax problem under the high-tuition scheme as defined in Problem (22) where the competition incentive is absent. First, we find that switching to the high-tuition college admission scheme leads to a significant decline in parental spending on private tutoring, as suggested by Figure 5. Consequently, only around 4% of households invest above the threshold $\bar{i}$, compared to 52% in the benchmark competition case. Furthermore, we find that any positive private tutoring tax (coupled with public investment subsidies to maintain a balanced government budget) results in a welfare loss relative to scenario without government intervention. This further confirms that the presence of the competition incentive justifies the need for taxing private tutoring expenditures. Because of the decreasing returns to scale nature of converting pre-college human capital into productive human capital (i.e., $\lambda = 0.46 < 1$), when pre-college human capital levels are relatively low because of low investment, the effective return on pre-college human capital is relatively high. Consequently, even a very small tax on private investment leads to welfare losses.

6.3 Allowing λ to Vary by Education Level

In this section, we allow the conversion parameter λ to vary by education level in our benchmark model in Section 2, i.e., $\lambda^s, s \in \{col, ncol\}$. Since we do not have direct estimates for $\lambda^s, s \in \{col, ncol\}$ from data, we internally calibrate these parameters to target the regression coefficient α_1 in Equation (20), as we do in Section 3.3, maintaining the difference between λ^{col} and λ^{ncol} as estimated in Abbott, Gallipoli, Meghir, and Violante (2020). We recalibrate the remaining model parameters using the same method as in Section 3.3, resulting in $\lambda^{col} = 0.79$ and $\lambda^{ncol} = 0.51$. This implies that college education is complementary to child ability, as high-ability children are more likely to attend college and thus enjoy a higher return (λ^{col}) on pre-college human capital.

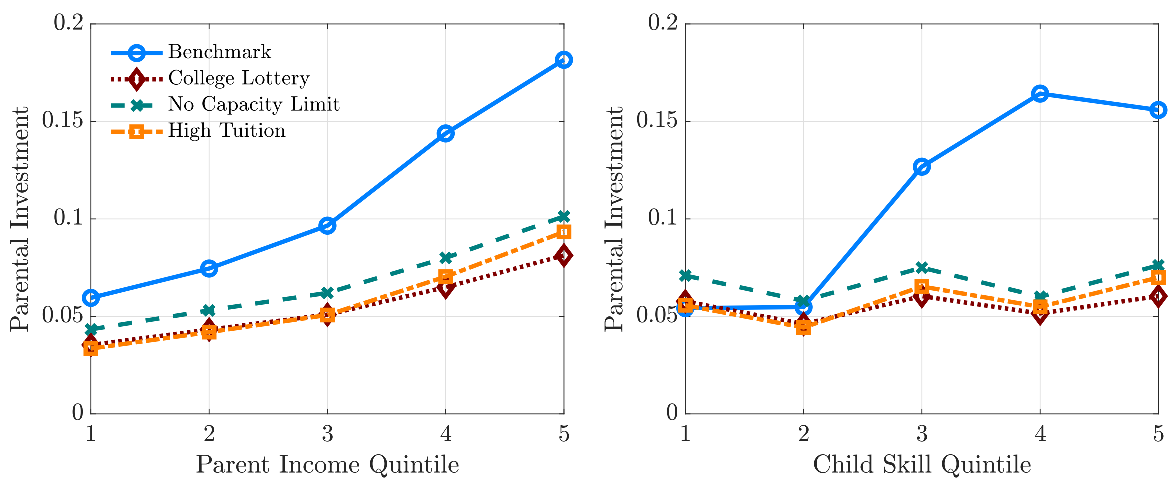

Figure 11: Parental Investment Patterns with Education-specific λ

Note: This figure displays parental investment at $j = 6$ (note the average household income is normalized to be one). The blue-circle-solid line represents the benchmark case. The violet-diamond-dotted line represents the random draw case. The green-cross-dashed line represents the case of relaxing capacity constraints. The orange-cube-dashed line represents the high tuition case.

First, we eliminate the competition incentive as in Section 4.1 and present the results on parental investment patterns in Figure 11. Even when the model features complementarity between college education and child ability, we are not able to generate non-monotonic patterns of parental investment with respect to child ability in the absence of the college competition incentive. In this extended version of our benchmark model, the competition incentive accounts for approximately 40-50% of parental investments (depending on how competition is eliminated) for an average household, which is lower than the 61-65% implied by our benchmark model. The primary reason behind this is that the extended model featuring heterogeneous λ has a higher conversion rate of pre-college human capital into productive human capital due to a higher weighted average λ.

Next, we examine the impact of private education investment tax (combined with linear public investment subsidies) within the model featuring heterogeneous $\lambda^s, s \in \{col, ncol\}$. The results are reported in Column (iv) of Table 8. The optimal tax rate is 15%, yielding a long-run welfare gain of around 0.1%. Allowing the conversion parameter to vary by education level does not change our conclusion on how competition for college admissions influences our perspective on child development policies (taxing vs. subsidizing investment). As the competition incentive is weaker in this extended model compared to the benchmark, the optimal tax rate and welfare gains are lower than those in our benchmark model.

7 Conclusion

College is widely viewed as an engine of the intergenerational mobility of socioeconomic status. The college selection stage links early childhood development factors, such as innate child ability and parental investment, to adult labor market outcomes. Therefore, college admission practices can potentially shape parents' incentives to invest in children's human capital and subsequent child achievement, and have broad implications for child development policies.

In this paper, we examine a scenario where college seats are limited and capacity cannot be easily expanded. This situation not only mirrors the reality in developing countries like China and India, where the supply of four-year colleges is not very affluent, but also applies to advanced economies like the U.S., where elite university spots are scarce. In such an environment, students compete for college admission based on their human capital, which is partially determined by parental investments. Using Chinese household survey data, we find that the human capital acquired for gaining college admission only partially translates into labor efficiency units, and the conversion rate diminishes as pre-college human capital rises. We offer a novel identification strategy that leverages the non-monotonicity of parental investment with respect to child ability, a distinct empirical pattern observed in Chinese data, to identify the parameter that governs the conversion rate. Our quantitative results further show that, for an average household, more than 60% of parental investment is driven by the competition incentive.

We deliver two important policy insights. First, under an exam-oriented college admission scheme with inelastic college supply such that the competition incentive for parental investment is very strong (due to low conversion rate), taxing private education investment can be welfare improving in that it balances curbing the competition incentive and minimizing human capital losses. Moreover, the competition incentive generates a high complementarity between child innate ability and parental investment, regardless of parental income. This suggests that child ability could be an equally important factor, if not more so, in the analysis of child development and corresponding policy guidance, beyond the factor of parental income emphasized by existing studies in the U.S. context. Consequently, policymakers may need to pay greater attention to supporting children with low abilities, who may persistently face disadvantages as a result of receiving limited private investment.

Our model could be extended by allowing labor efficiency units to depend on multiple skill types and enabling parents to invest in their children's pre-college human capital across various dimensions. In test-score-based college admission systems, such as those implemented in China, India, Japan, and Korea, the competition incentive may cause parents to overemphasize their child's cognitive abilities while underinvesting in other skill types. These investment distortions may lead to sizable permanent income and welfare losses, and may have crucial macroeconomic implications on innovation, productivity, and growth. We leave a more thorough and rigorous analysis of these issues for future research.

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Online Appendix
The Macroeconomic Consequences of Competition for College Admissions

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A Data and Measurement

A.1 Parental Investment Patterns: China v.s. the US

In the main text, we examine how parental monetary investment depends on child cognitive skills after controlling for parental income quintiles. We argue that the dramatic increase in parental investment when a child's skill reaches a certain threshold in the distribution is a unique feature observed in Chinese data. We document parental monetary investment patterns using the Child Development Supplement (CDS) of the Panel Study of Income Dynamics (PSID) and follow the procedures in Lee and Seshadri (2019).

As shown in Figure A1, parental investment with respect to child cognitive skills in the US exhibits significantly different patterns compared to those of China. US households invest slightly more in children with low cognitive skills relative to their labor income.

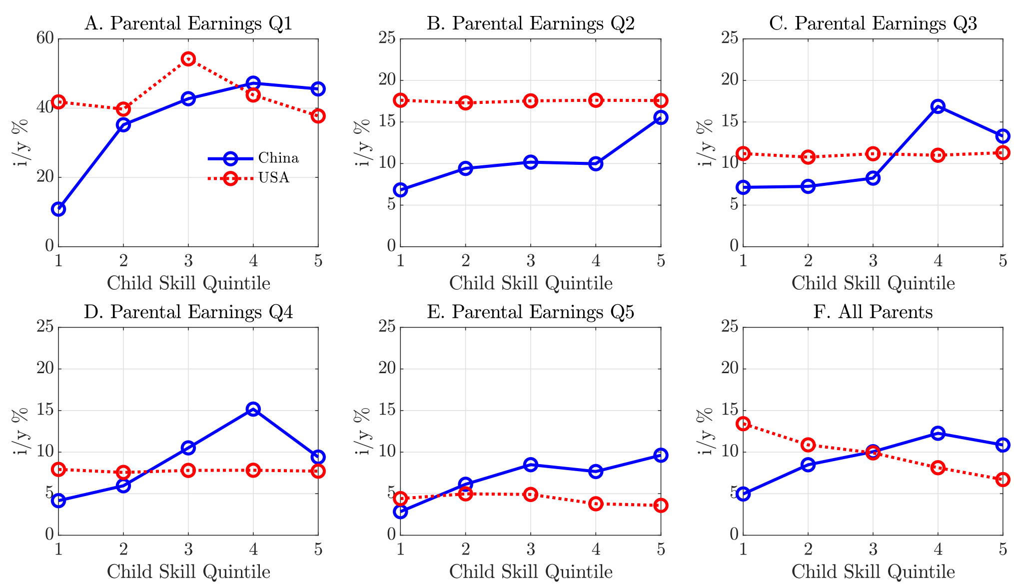

Figure A1: Parental monetary investment patterns: China v.s. the US

While parental time investment plays a larger role in a child's human capital development in the US, the striking difference in monetary investment patterns between the two countries suggests varying parental objectives. Chinese parents invest in their children's human capital to enhance their chances of college admission, leading to greater investment for those near the admission threshold. In contrast, US parents invest to compensate for children with lower abilities, thereby reducing the ability gap. Consequently, parental investment amplifies the innate ability gap in China, while diminishing it in the US.

A.2 Age Profile of Earnings

We use the the China Health and Nutrition Survey (CHNS) to estimate the age profile of earnings by education A_j^s, $s \in \{col, ncol\}$, and report the results in Figure A2.

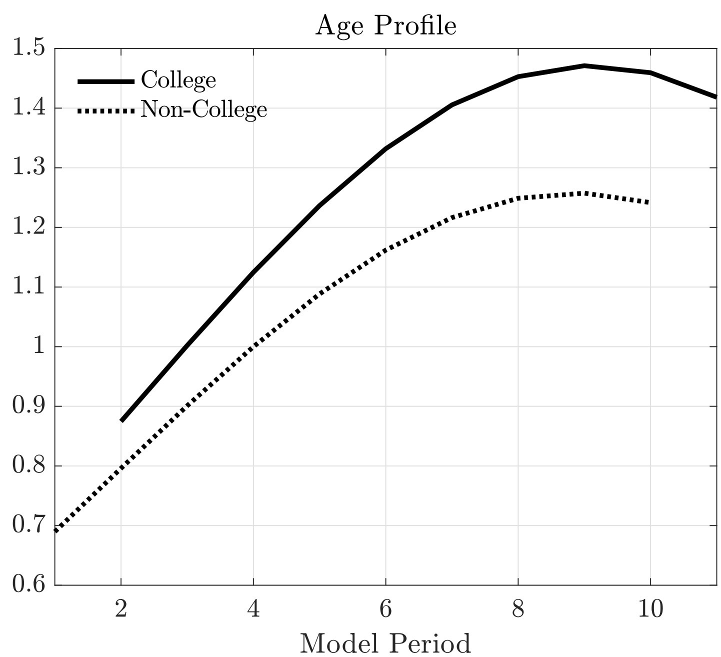

Figure A2: Age Profile of Earnings

Note: The x-axis is for model periods j. The value reported on y-axis is $exp(A_j^s)$, $s \in \{col, ncol\}$. Note that the average household income is normalized to be one.

A.3 Public Education Expenditures

We utilize the China Education Finance Statistical Yearbook (CEFSY) to gather information on public investment in both pre-college and college education, and present the results in Table A1. The currency unit used in the table is RMB *yuan*.

Table A1: Public Education Expenditures

Panel (A)	Child age	Model period	Public expenses	$g/\text{avg}(i)$
Primary school	6-11	5, 6	7,023	1.68
Middle school	12-14	6, 7	9,543	2.49
High school	15-17	7	8,722	1.98
Four-year college	18-21	8	18,663	3.11
Panel (B)	**Child age**	**Model period**	**Average earnings**	$g/\text{avg}(y)$
Stage 1	6-9	5	47,059	0.15
Stage 2	10-13	6	39,914	0.21
Stage 3	14-17	7	37,159	0.25

B Welfare Measure

Let $\overline{\text{CE}}_s$ measures the fixed proportional consumption transfer to all newly-independent individuals with education s and pre-college human capital h_p in the benchmark economy such that average utility is equal to that in the alternative economies. It reads

$$\int V_1^{\text{BM}}\left((1+\overline{\text{CE}}_{ncol}) \times c_1^{\text{BM}}(ncol, h_p)\right) d\Psi^{BM}_{ncol} = \int V_1^{\text{AL}}\left(c_1^{\text{AL}}(ncol, h_p)\right) d\Psi^{AL}_{ncol}$$

$$\int V_2^{\text{BM}}\left((1+\overline{\text{CE}}_{col}) \times c_2^{\text{BM}}(col, h_p)\right) d\Psi^{BM}_{col} = \int V_2^{\text{AL}}\left(c_2^{\text{AL}}(col, h_p)\right) d\Psi^{AL}_{col}$$

where V_j is the lifetime value function and c is the consumption allocation starting from period j and individual state (s, h_p), where Ψ_{ncol} and Ψ_{col} are the initial distribution of households (over h_p) whose education level is non-college and college, respectively, and the superscripts indicate the relevant economy (BM denotes benchmark and AL denotes alternative economies).

Using the function form of utility function $u(c) = \log(c)$, we can compute $\overline{\text{CE}}_s$ as:

$$\overline{\text{CE}}_{ncol} = \exp\left(\frac{\int V_1^{\text{AL}}(ncol, h_p)\, d\Psi^{AL}_{ncol} - \int V_1^{\text{BM}}(ncol, h_p)\, d\Psi^{BM}_{ncol}}{\sum_{j=1}^{15}\beta^{j-1}}\right) - 1$$

$$\overline{\text{CE}}_{col} = \exp\left(\frac{\int V_2^{\text{AL}}(col, h_p)\, d\Psi^{AL}_{col} - \int V_2^{\text{BM}}(col, h_p)\, d\Psi^{BM}_{col}}{\sum_{j=2}^{15}\beta^{j-2}}\right) - 1$$

where $V_j(s, h_p)$ is the value function of a household of period j in state (s, h_p), and the superscripts indicate the relevant economy. The aggregate welfare change equals the sum of the education-specific welfare changes:

$$\overline{\text{CE}} = \overline{\text{CE}}_{ncol} + \overline{\text{CE}}_{col}$$

Note that in Section 5.1, when we compute the short-run welfare changes from regulation polices, the human capital distribution of parents (Ψ_{ncol} and Ψ_{col}) are exogenously estimated and do not vary due to policy changes, as discussed in Section 3.2. In Section 5.2 and Section 6, when we compute the long-run welfare changes, these two distributions are endogenously generated from the stationary equilibrium.

In Section 5.1, we examine the short-run distributional welfare effects of private tutoring ban across heterogeneous families, as shown in Table 6. The main welfare measure we consider is the cumulative household utility from period 5 (the time of the child's birth) to period 15 (the final period of the life cycle). To do so, we first compute consumption equivalent variation CE for an j-period-old household whose state variable is $x_j = (s, a, h_p, z, h)$ as the percentage change in consumption at all future dates and states required to make her indifferent between the two economies:

$$V_j^{\text{BM}}\left((1+\text{CE}_j(x_j)) \times c_j^{\text{BM}}(x_j)\right) = V_j^{\text{AL}}\left(c_j^{\text{AL}}(x_j)\right)$$

The individual-level consumption equivalent variation at period 5 is defined as

$$\text{CE}_5(x_5) = \exp\left(\frac{V_5^{\text{AL}}(x_5) - V_5^{\text{BM}}(x_5)}{\sum_{j=5}^{15} \beta^{j-5}}\right) - 1$$

Next, we categorize households into different groups based on parental income and child innate ability, and then calculate the average welfare changes for all individuals within each group.

C Additional Results on Quantitative Analysis

C.1 More Results on Model Performance

Table A2: Additional Non-targeted Moments

Moment	Data	Model
(A) Parental investment		
Avg. parental investment to income (col parent)	0.19	0.17
Avg. parental investment to income (non-col parent)	0.12	0.13
SD of log parental investment	1.40	0.81
Regression coefficient i/y on $\log(y)$	-0.06	-0.05
Regression coefficient $\log(i)$ on $\log(h_p)$	0.84	1.05
Regression coefficient i/y on $\log(h_p)$	0.04	0.06
(B) Intergenerational correlation		
Regression coefficient $\log(h_8)$ on $\log(h_p)$	0.22	0.26
Correlation coefficient $\log(h_8)$ on $\log(h_p)$	0.33	0.37

Table A2 presents additional empirical moments not targeted in the calibration, along with their model-generated counterparts. Overall, our model aligns well with the data. Particularly, we evaluate whether the model accurately captures how parental investment depends on parent characteristics, including education, human capital, and labor income, as observed in the data. Our model successfully reproduces these aspects, although it underestimates variations in parental investment. Moreover, we find that our model well replicates the results on how child development outcomes depend on the parent human capital observed in the data.

C.2 Re-calibrated Parameters when Varying λ

In Table A3, we report the recalibrated parameters when λ is varied. Note that the only empirical moment we are not targeting when λ deviates from the benchmark estimated value 0.46 is the regression coefficient α_1 in Equation (20), which captures how parental monetary investment i_j depends on child cognitive skills in the current period h_j after controlling for parental income y_j and year fixed effects. In the extended model with education-specific λ specified in Section 6, we still target α_1.

Table A3: Internally Estimated Parameters by λ

Description		$\lambda = 0.20$	$\lambda = 0.46$	$\lambda = 1.00$	$\lambda^{col} = 0.79$ $\lambda^{ncol} = 0.51$
(A) Parameters					
Discount factor	β	0.89	0.89	0.90	0.90
Altruism	ν	0.44	0.37	0.26	0.35
Age-dependent efficiency of skill production	θ_5	3.59	3.14	2.88	3.12
	θ_6	2.45	2.09	1.86	2.07
	θ_7	1.99	1.69	1.51	1.68
Age-dependent weight on parental investment	η_5	0.33	0.41	0.51	0.40
	η_6	0.26	0.34	0.45	0.33
	η_7	0.21	0.30	0.44	0.29
Weight on current child skill	α	0.47	0.64	0.75	0.65
Dynamic complementarity	γ	0.65	0.42	0.34	0.40
Elasticity of substitution between i and g	μ	0.21	0.19	-0.13	0.16
Persistence of IG skill	ρ_h	0.09	0.07	0.06	0.07
Avg. child skill endowment (college)	μ^c	0.29	0.32	0.35	0.32
Avg. child skill endowment ((non-college)	μ^n	0.33	0.34	0.35	0.34
SD IG genetic shocks (college)	σ_h^c	0.11	0.11	0.12	0.11
SD IG genetic shocks (non-college)	σ_h^n	0.12	0.12	0.12	0.12
Persistence of labor productivity shocks	ρ_z	0.72	0.71	0.65	0.71
SD of labor productivity shocks	σ_z	0.22	0.21	0.19	0.21
College labor intensity (firm production)	ϕ	0.30	0.29	0.27	0.28
College admission function shifter	ζ	-0.05	-0.06	-0.09	-0.07
(B) Regression coefficient α_1					
Data		1.14	1.14	1.14	1.14
Model		2.21	1.14	0.22	1.14

C.3 More Results on Decomposing Parental Investment by λ

Based on the re-calibrated model parameters discussed in the previous section, we also report how decomposition results vary by λ and stage of child development in Figure A3.

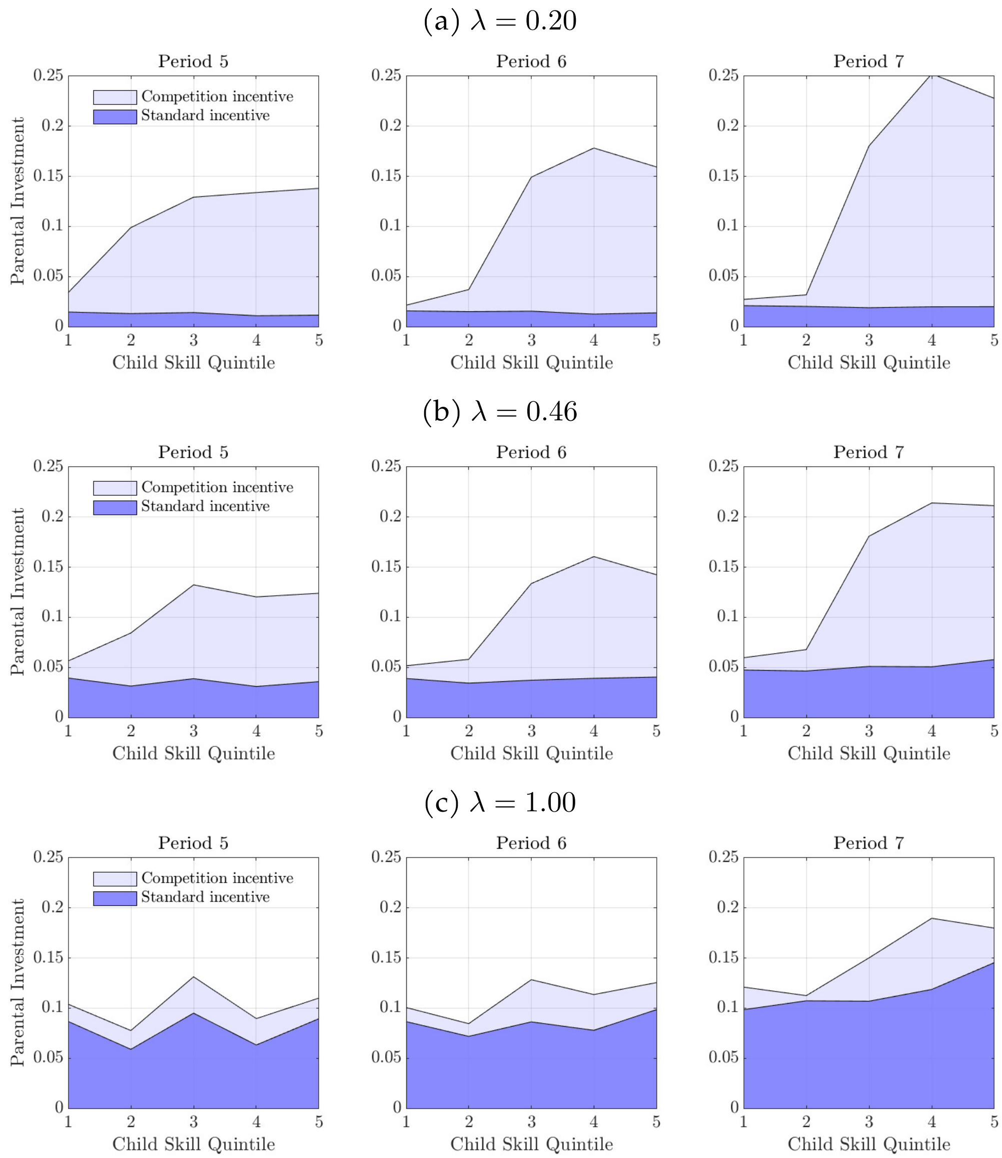

Figure A3: Sources of Parental Investment by λ

C.4 Conversion Rate

In Figure A4, we report that with our baseline value of $\lambda = 0.46$, how pre-college human capital h_p translates into production human capital h_k. Note that $h_k = h_p^{\lambda}$. The conversion rate is defined as the first order derivative of h_k with respect to h_p, i.e., $\lambda h_p^{\lambda-1}$. As we can see, the conversion rate declines as the stock of pre-college human capital rises.

We further check how pre-college human capital h_p translates into production human capital h_k with different levels of λ, and report the results in Figure A5. As we can see, the higher the λ, the higher the conversion rate at any given level of h_p over the entire

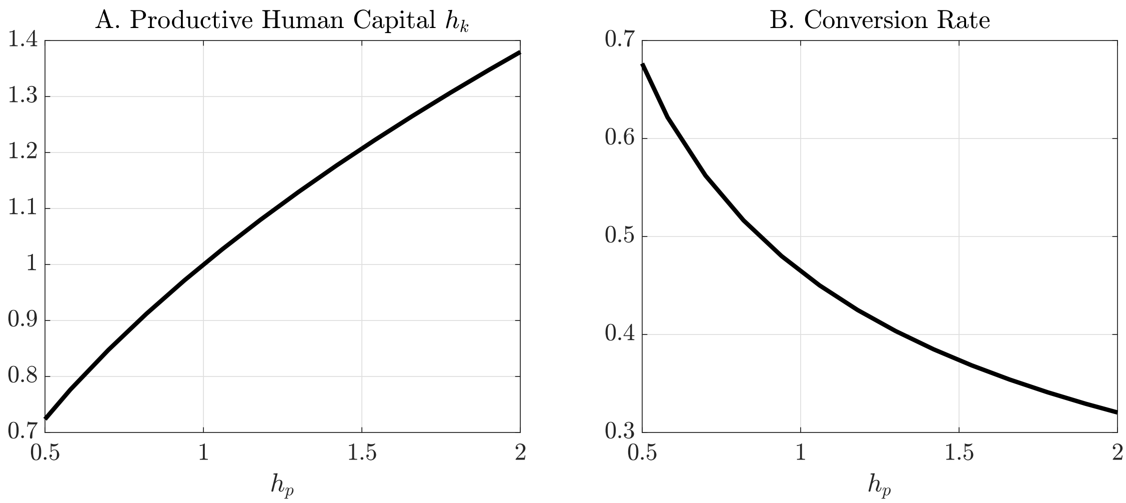

Figure A4: Conversion rate of h_p to h_k in benchmark model

distribution of pre-college human capital.

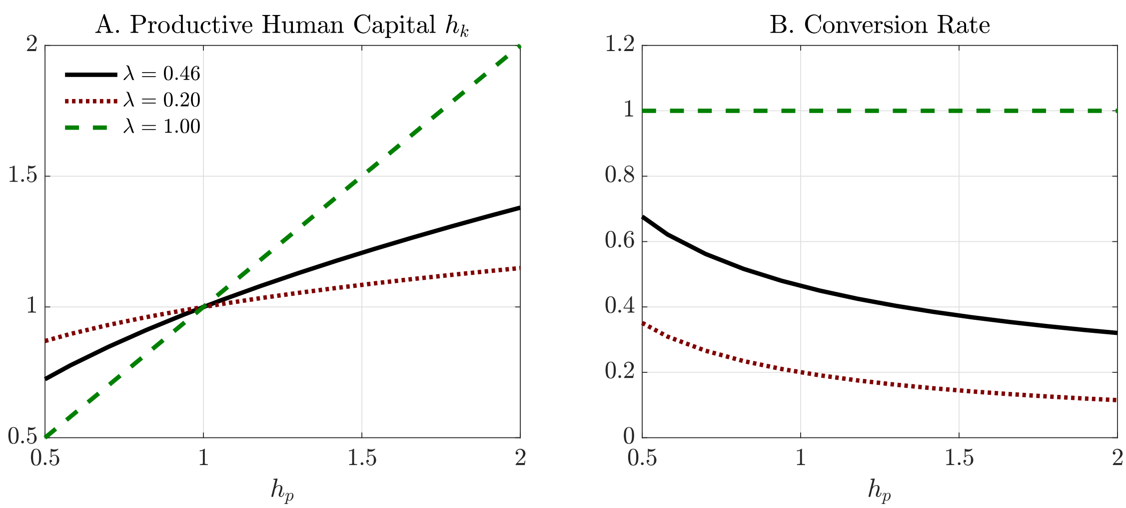

Figure A5: Conversion rate of h_p to h_k by λ

C.5 Details on eliminating competition incentive

We report more detailed results regarding Figure 5 in Table A4.

Table A4: Parental Investment: Compare with No-Competition-Scenarios

(i) Parent Earnings	Q1	Q2	Q3	Q4	Q5	All
Benchmark	0.059	0.074	0.097	0.139	0.178	0.109
College Lottery	0.027	0.033	0.039	0.050	0.063	0.043
(% change)	(-54.6%)	(-55.1%)	(-59.5%)	(-63.8%)	(-63.4%)	(-61.0%)
No Capacity Limit	0.023	0.029	0.035	0.045	0.057	0.038
(% change)	(-60.1%)	(-60.3%)	(-64.1%)	(-67.6%)	(-67.9%)	(-65.2%)
High Tuition	0.028)	0.035	0.040	0.048	0.058	0.042
(% change)	(-51.7%)	(-53.5%)	(-58.9%)	(-65.2%)	(-67.4%)	(-61.8%)
(ii) Child Skill	Q1	Q2	Q3	Q4	Q5	All
Benchmark	0.052	0.058	0.134	0.161	0.143	0.109
College Lottery	0.044	0.037	0.043	0.044	0.045	0.043
(% change)	(-14.1%)	(-36.2%)	(-68.0%)	(-72.7%)	(-68.4%)	(-61.0%)
No Capacity Limit	0.039	0.034	0.037	0.039	0.040	0.038
(% change)	(-24.6%)	(-41.0%)	(-72.2%)	(-75.7%)	(-71.7%)	(-65.2%)
High Tuition	0.044	0.037	0.042	0.042	0.043	0.042
(% change)	(-15.2%)	(-35.0%)	(-68.3%)	(-73.6%)	(-69.8%)	(-61.8%)

Note: We categorize parental income and child cognitive skills for the 10-13 age group into quintiles, and report the average parental investment for each quintile in the benchmark, high-tuition scenario, and no-capacity-limit scenario, respectively.

C.6 More results on policy implications

C.6.1 Underground case of the short-run impacts of private tutoring ban

To address the potential unintended consequence of the ban driving the private tutoring sector underground and enabling only wealthy parents to hire tutors, we double the price of parental investment for any amount exceeding the cap. This effectively simulates a 200% proportional tax rate on private tutoring, with the tax revenue remaining unutilized. We report the results in Table A5.

Pre-college human capital losses and welfare As shown in Table A5, parents' average consumption increases by around 1%, and children's pre-college human capital (at the age of entering college) declines by around 5%. Consequently, children's expected lifetime earnings decrease by an average of around 2%. Welfare, measured in terms of consumption equivalent utility of the parent generation, is affected by the private tutoring ban in two ways. On one hand, reduced spending on children's education can increase parents' consumption and welfare, and improve investment efficiency for those who overinvest due to competition incentive. On the other hand, a decline in children's expected lifetime utility reduces parents' welfare since they care about their children's well-being for altruistic reasons. Overall, parents' average welfare increases by 0.4%. The magnitudes of the change in all variables are smaller compared to the complete private tutoring sector shut-down case.

Inequality and mobility As shown from the last two blocks of Table A5, doubling the price of private tutoring also significantly reduces inequality and intergenerational persistence in the child generation, but the magnitudes of the decline are smaller compared to the complete private tutoring sector shut-down case.

Table A5: Short-run Aggregate Outcomes for Private Tutoring Ban in Underground Case

	Benchmark	Ban	Underground
(i) Aggregates			
Parental investment	0.12	-39.18%	-29.41%
Effective investment	0.17	-10.66%	-9.15%
Pre-college human capital	1.20	-5.78%	-4.97%
Expected lifetime earnings	1.27	-2.83%	-2.42%
Consumption	0.92	+1.55%	+1.16%
(ii) Welfare			
Consumption equivalent	-	+0.49%	+0.38%
(iii) Inequality			
Var log lifetime earnings	0.14	-8.10%	-6.73%
Var log consumption	0.15	+7.74%	+5.63%
(iv) IG correlation			
Human capital	0.37	0.26	0.29
Education	0.23	0.06	0.13
Permanent income	0.40	0.23	0.29

C.6.2 More results on long-run impacts of optimal private tutoring tax

We find a rate of 30% maximizing the long-run ex-ante lifetime utility in the case of private tutoring tax combined with a linear subsidy on pre-college public investment, as shown in Figure A7.

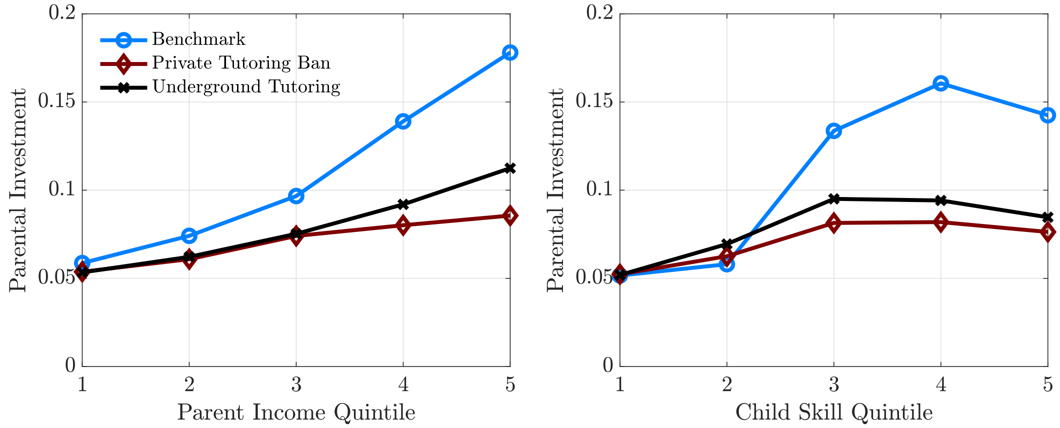

Figure A6: Investment with Family Heterogeneity with Underground Case

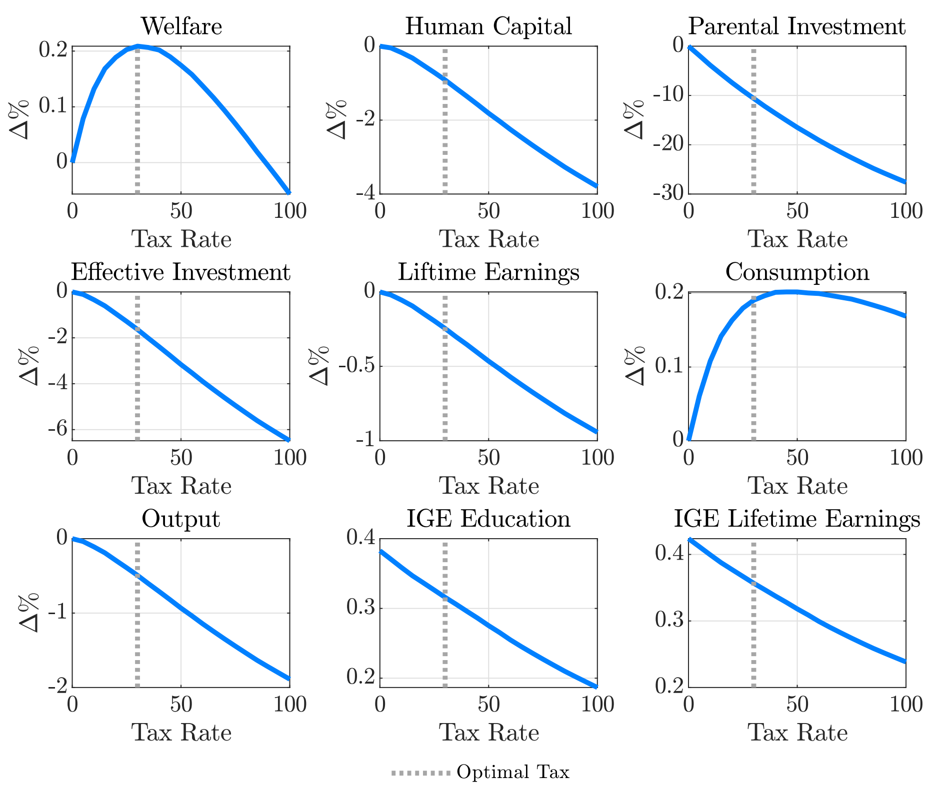

Figure A7: Optimal Private Tutoring Tax in Long Run